Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 253 MARCH, 1952 No. 3

SCIENCE IS NOT ENOUGH *

BY

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While, on the one hand, we are frequently warned that we are experiencing a "cultural lag" because science and technology (we really mean industrial progress) has progressed far more rapidly than our abilities to assimilate its contributions in our social life, there is, at the same time, a wide-spread realization of the dominating role which engineering and engineered industry have come to play in the advance of modern civilization. Our progress in peace and our strength in war are dependent upon our continued technological and industrial leadership. Indeed such progress is not simply desirable, it is essential if our civilization is to survive in this modern world. It is as if some new evolutionary force has come into the life of man, and, whether we believe it to be good or evil, it inescapably impels us forward—it has a grip on humanity which we cannot relax, which we must, in fact, continue to further and develop.

Certainly in this picture of man in the hand of God, or fate, or inescapable destiny—as you will—the role of science in engineering plays a major part. Yet here we find not only popular misunderstandings but viewpoints and policies stemming from these erroneous ideas which threaten our continued leadership both in engineering and in science, a leadership so vital to the survival of Western man.

* While this paper is based on an introductory address delivered at a conference sponsored by the Thomas Alva Edison Foundation last November, the author is indebted for the title to his friend Mr. Gano Dunn, President of the J. G. White Engineering Corporation and Chairman of the Board of Trustees of The Cooper Union. It is interesting to note that Mr. Dunn was one of three members recently honored by The Franklin Institute upon their completion of half-a-century of continuous membership.

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There have, unfortunately, been few attempts to appraise with impartiality this liaison between science and engineering. On the one hand, the natural sciences have developed in the last quarter century a program of propaganda which, however essential in securing the support which research in science not only deserves but must have, has resulted in establishing in the popular mind the notion that engineering is merely a commercialization of science—is, in fact, simply "applied science." Many people have assumed, therefore, that expanded research in science is all that is necessary to insure our continued technological and industrial supremacy. "The pure science of today," it is said, "becomes the applied science of tomorrow." On the other hand, engineers have been too busy "doing" to worry about what they regard as largely an "academic" question.

Probably the basic fallacy in this campaign of misunderstanding has been the statement, made and spread by many able scientists, that engineering is simply an outgrowth of and development from modern science, born of scientific research and still completely nourished by its parent. Important as the present liaison between science and engineering is, no more completely untruthful and dangerous statement could be made.

As in the case of medicine, engineering as we know it today is the modern version of an ancient practical art. Medicine stems from the ancient art of healing, engineering from the practical art of construction, which, having beginnings at least fifty centuries ago, is at least as old as civilized life itself. Mining and metallurgy have origins almost equally ancient while our modern chemical industries spring from the ceramic, dye, drug, leather, cement and other arts of Ancient Times. Egypt is recalled also as the Mother of the Mechanical Arts. These origins are far more venerable than any claims which may be made for science. It has been only in the last century, as a matter of fact, and in one field, electricity, that a new branch of engineering has been created, largely as a result of basic scientific discoveries.

Furthermore, the rise and evolution of engineering in strength and power has been concurrent with that of Western Civilization. Engineering, like the stream of Western life, had its beginnings with the dawn of civilization in the fertile valleys of the Nile in Egypt and the Tigris-Euphrates in Mesopotamia. Greece drew together these earlier advances and the Greeks became the first great harbor and city builders, developing techniques which were passed on to Rome. Rome, in turn, spread over the entire Ancient World standards, especially of urban life,—aqueducts, roads, bridges—which the world was not to know again for over a thousand years. Even in the Dark and Middle Ages, following the collapse of the Western Roman Empire, while ancient culture was temporarily lost, these material accomplishments remained

as mute reminders of what man had once done and could again venture to undertake.

The Middle Ages is one of the most interesting periods in the history of engineering. Its fruits were more fully developed in the Italian Renaissance and were imported to France through the Franco-Italian wars. For over two centuries—the 17th and 18th—France held the leadership not alone of Western culture but in engineering. With the French Revolution, however, this era came to an end. The Revolution also marked the culmination of the first great era of engineering development.

We frequently speak of Ancient and Modern History and, similarly, the terms Ancient and Modern Engineering have been used. Actually, however, Ancient Engineering did not cease with the collapse of the Roman Empire in the 5th Century. France's major engineering accomplishments in these two or more centuries when she carried the torch of Western Civilization, were in the further development and refinement of ancient engineering activities and methods. As it had been in Roman days, so in pre-revolutionary France the emphasis was on the practical art of construction, on roads and bridges, on water supplies and harbors, and on government, public works and military engineering.

Similarly, the materials which these French workers had at hand were the same materials with which their earlier Roman comrades had labored—stone and timber, brick and lime. A real age of metal was yet to come; metals were too costly to use for other purposes than tools, weapons and fastenings. The power sources the French turned to were also the same human, animal, wind and water man had used almost from the dawn of civilized life. Their technical knowledge was still that of a remarkable qualitative empiricism—rules-of-thumb, if you will—derived from centuries of experience, close contact with doing as well as planning.

We must not belittle their accomplishment. The Middle Ages had produced man's greatest triumph in the use of stone, that bird-cage of skeleton-stone construction, the Gothic cathedral. These later French engineers pioneered in modern canal and road construction and brought to its final perfection, after two thousand years of development, the stone arch bridge. Engineering was, as it had long been, a powerful instrumentality in the support and progress of civilized life.

But all this French engineering was in the domain of governmental construction and left, almost untouched, the application of engineering methods in the great area of the mechanical arts, in manufacturing and industry. French manufactures were still largely small, rural activities plus a few urban works, unprogressive and dominated by guild restrictions. In short, while engineering had been a potent instrumentality in the material advance, especially of Western urban life, its possibilities and potentialities in the development of a new and more progressive

economy and culture based on engineered-industry were yet to be recognized. This fundamental change to a new engineering epoch took place in Britain. Across the English Channel, a new era of revolutionary import had been developing during the 18th Century.

Britain had solved her political problems and established her own peculiar form of democratic-monarchy in the 17th Century. Enclosure, from beginnings under Elizabeth, was bringing in a change in agriculture which was to lead, not to the creation of a multitude of small owners as it so largely did on the continent, but to the birth of a free laboring class. The possibilities of the older, agricultural-home economy were reaching their end. Necessity, it has been said, is the Mother of Invention. New and fruitful employment had to be found for this new group. Agricultural reform and the Industrial Revolution went hand-in-hand in Britain.

We need not follow through the details of this development to understand its impact on modern life. Beginning in the earlier 1700's the iron industry was reborn through the application of the coal-iron process, that is coke smelting. Kay with his fly-shuttle, later Arkwright and others mechanized the textile industry. By the time of the American Revolution Watt had added steam power and by the close of the century it was turning the wheels of British industry. Early in the next century, it revolutionized transportation. A completely new form of culture, a new way of life, was in process of creation. It was on this new culture, this new way of life, that Britain's commanding world position of the Victorian Era was based. At the time of the British Exposition of 1851, Thackery had written:

> "Look yonder where the engines toil:
> These England's arms of conquest are, . . .
> With these she sails, she weaves, she tills,
> Pierces the everlasting hills
> And spans the seas."

Machines of iron driven by steam were not only supplanting man power but making it possible for a few men to do in a few hours what it had taken many men long days of toil and sweat to accomplish. The paradox of greater production with less labor was becoming an accomplished fact. An era in which man was released from physical toil was, at long last, beginning to dawn.

Adam Smith, the Father of Modern Economics who wrote *The Wealth of Nations* as this vital change was in progress, saw in the division of labor, in specialization, the key to the new era. He tells the story of the blacksmith, a general worker in iron, who, when he turned to making nails, could turn out but a hundred or so a day whereas an apprentice who made nothing but nails might produce up to two thousand. But he noted also that such specialization inevitably led to the de-

velopment of specialized equipment, new tools and machines, which further increased production. As this new way to wealth developed, agriculture in Britain declined and the British economy became one of the exchange of manufactured goods not only for raw materials but for the food necessities of life. As the noted British economist Clapham puts it: "in these wheels of iron and shriek of escaping steam lay for Britain not only her power and wealth but her very existence." Income from industry was supplemented, of course, by the returns of shipping, foreign investments and other "invisible" sources of revenue, but industry was the base for Britain's economy.

As is well known, it was from this ground-work in Britain that our present industrial civilization in the United States has been built. In spite of our very interesting and notable industrial innovations in the 19th Century, however, American industrial growth has been primarily a 20th Century development—a matter of the past half century. The dominating problem of 19th Century America was transportation and this meant the railroad. Remarkable as were our earlier accomplishments—our inventions and the scale of our efforts—before 1900, they seem, in the light of our subsequent expansion, to have been merely a making ready of the way.

For example: In 1900, industry in the United States was still largely confined to the New England and North Central States. Some fourteen states, occupying less than a sixth of our area, supported almost half our population and enjoyed over 60 per cent of our national income. Today no state in the union produces less than twenty million dollars of manufactured products and only four less than one hundred. Between 1900 and World War II our national production increased between five and six fold and, as our population had less than doubled, there were about two to three times the goods and services at hand for twice the number of people that were available fifty years earlier. Furthermore, this flood of material wealth, perhaps a third of the world's production, we share more widely than in any age in history or any nation on earth. Our task has been that of finding useful productive and remunerative employment not only for a population which has doubled in the last fifty years but on an ever-increasing scale and for an active nation of workers. This task modern American technology and industry has accomplished.

The third phase in engineering history dates back to the mid-19th Century. "Within the last hundred years," observes Professor A. North Whitehead of Cambridge and Harvard, "a developed science has wedded itself to a developed technology and a new epoch has opened." This statement not only epitomizes this third phase, or era, in the evolution of engineering but raises the question: What part has science played in this remarkable later American advance—the American Industrial Revolution of the 20th Century?

It seems clear that the general scope and character of engineering was established long before this wedding which Professor Whitehead notes. It was a *developed* technology which was wedded to a *developed* science. In other words, for forty-nine of the fifty centuries of recorded history engineering and science went their own, almost wholly separate and unrelated ways. There are, however, within every practical art the germs of a practical science. Beginning in France in the late 18th Century there was a marked interest in the search for a more scientific technique in the design of engineering works. For example: Man had built stone arch bridges for close to two thousand years, and, through an intuitive feeling for structure reinforced by these ages of experience, gained some remarkably acute, although purely qualitative, understandings of how arches acted and the qualities which were essential to their stability. Through the new applied-mechanics approach this hard-won knowledge was confirmed and extended but, especially, gradually increased in quantitative accuracy. More economical design resulted—not only bigger but better bridges. The same story has been followed in the development of the truss and other bridges, the steam engine, the electric light, the aeroplane and a host of other devices. In other words, these products did not stem from scientific research but, without exception, they have been improved or perfected—a better product at less cost has resulted from the application to their analysis and design of the more exact and quantitative measurements and methods of science.

Mr. H. W. Martin of the famous Bell Laboratories has most interestingly described the sequences in this process in an article marking the seventy-fifth anniversary of the telephone. While this instrument is generally regarded as a product of scientific research, Mr. Martin makes it clear that invention came first. The telephone, of course, relies on the electro-magnet but the basic idea of the electric transmission of speech was a vision of Bell's rather than an electro-magnetic inspiration. The first step was the invention of the device—the marriage of an idea with the means of its realization. The second was the development of the device into a practical, useful and manufacturable, instrument. Only then did science and theory begin to enter the story. What would be the effect of this or that change or modification? Even here, however, this query remained in the qualitative stage until measurements could be devised which made it possible to replace "cut-and-try" methods of improvement with actual measurement and quantitative, definite procedures. The result: An improved device of better quality produced at a lower cost.

One of the widespread misunderstandings of the present day—and it is one which the propaganda of science has done much to promote and broadcast—is the notion that engineering is largely invention and that the major role of science in engineering has been in invention. As

a matter of fact new inventions play a far less important role in engineering than is popularly supposed. It is probably safe to say that over ninety per cent of engineering consists in doing better what has already been done many times before—better bridges, roads, water supplies, engines, machines, etc. This is not to say that all these things are not constantly improved through minor changes and inventive modifications. On the other hand, the creation of entirely new products, services and industries through invention is exceptional in engineering rather than an activity to which—as the public has been led to believe—the great bulk of engineering effort is directed. The major, the outstanding, role of science in engineering has, thus, been that of the improvement of previous practice rather than the creation of new services.

This is illustrated in the telephone sequence outlined above. Few earlier, basic inventions have stemmed directly from scientific discovery, as we have noted, but without exception the application to their development of what have become known as the methods of science has resulted in continued improvement and progress. But, it should also be made clear, the discoveries of science have increasingly provided a means of realizing an inventive idea. It was the electro-magnet, for example, that provided the answer to Bell's vision. The further question may be asked: What part does science play in the birth of the idea itself?

Here again there can be no doubt that some scientific discoveries have made possible the conception of inventive ideas. Faraday's discovery of magnetic induction, for example, undoubtedly led to the thinking and inventions which created the electrical power industry. But the notion that the dynamo was an immediate, direct and obvious development—a mere commercialization—of Faraday's discovery is sheer nonsense. It took at least fifty years to make from this discovery a practical dynamo and this would have been little more than a scientific toy if the parallel developments of electric light and the electric motor had not created a use for this machine—a market for electricity. Faraday's discovery triggered-off, so to speak, a long chain of developments but without these developments it would have been sterile and still-born. It's not knowledge but what we do with it that is of prime importance.

Probably there is no more striking example of the sterility of science alone as a prime factor in our industrial and economic life than that which has developed in Britain. From Faraday to Maxwell and Kelvin, from Darwin to Huxley and Tyndall, Britain has produced some of the greatest leaders in modern science. Yet in the last fifty years, years which have marked such an extraordinary era of material progress in the United States, Britain, the motherland of the Industrial Revolution, has been steadily declining in her industrial and economic position. If pre-eminence in science, as the propaganda of science so confidently

proclaims, is all that is necessary to continued industrial leadership and technological progress, why has Britain fallen into what has been described as technological and industrial stagnation and decay?

One would be unrealistic indeed to ascribe this British decline solely to the high-hat attitude of British science and the low estate to which engineering has fallen in Britain. To begin with, the British economy, based on the export of manufactured products and necessitating the import not only of basic materials but of essential food supplies, is clearly precarious and vulnerable to foreign competition. During the Victorian Period when Britain almost stood alone industrially speaking, and ruled the waves, all was well. But, with the turn of the century, it became apparent that Britain no longer held an industrial-export monopoly. The difficulty appears to have been that her industrial leaders did nothing about it. Adopting a self-satisfied, complacent attitude, she failed to follow up her earlier triumphs with continued developments and improvements—to realize that it is impossible to maintain a static position in a world based on technology and industry. Progress and change are not, as we have said, merely desirable—they are essential to survival. A stable, static economy is impossible—it is but the prelude to decline and decay.

This is not the place for an analysis of all the influences which have led to industrial and economic collapse in Britain. But certainly, one of the factors which has contributed to her unprogressive attitude has been what we have noted as the high-hat viewpoint of British scientists—the fact that science in Britain has succeeded in establishing the idea—as science in America seems bent on doing—that engineering is merely cheap, applied science and such applications can be left to those whose interests are vocational rather than professional and whose minds are directed solely to commercial pursuits—in short, given science, applications will take care of themselves.

Engineering is regarded as a "navvy," a laborers pursuit in Britain—it is not a recognized profession. Engineering education is still largely a matter of rather narrow vocational technical school instruction—it is not a recognized university activity. There also appears to be a notion that the engineer deals only with science, with the materials and the forces of nature. The fact that his task is production and that he must work with and direct men is ignored. A British dean of science at a great industrial center remarks:

"I do not look upon production as an engineering job at all; engineering in the true sense of the word is finished when the designs are made, the tools and jigs thought out and constructed on a department which does not concern itself with the production of the article to be sold. That is the men of brains and the men of skill are wholly concentrated in the drawing office and tool room; the rest of the workers are human automata."

Here is a washing of the hands of any connection with machine skills or engineered production, with the direction of labor. Here is a complete failure to realize that design is not an end in itself but merely a means to intelligent production.

The light has now begun to dawn, however, for, more recently, Sir Ewart Smith in a paper reprinted in *The American Scientist* clearly stated the truth, namely: "Any real basic knowledge which is evolved is, broadly and relatively quickly available to all, and it is therefore upon technological skill in application that the progress of industry and, consequently, the economic position of the nation will mainly depend." Scientists not only do not possess this skill—there is every reason why it should not be one of their interests—but they also seem to deny that even the technique of application is, in itself, a subject of special knowledge, study and research.

Scientific knowledge knows no national boundaries—the search for truth may be discouraged or retarded but cannot be stopped. Yet, in itself, knowledge is not power. Perhaps no more striking example may be cited in support of Sir Ewart Smith's statement than that of America's rise to industrial supremacy. Outstanding as we have been in the use of scientific knowledge in creating this industrial supremacy we have been, relatively speaking, but a minor contributor to the advance of basic, scientific knowledge. Even in the case of the atomic bomb the basic understandings were of European origin—our contribution has been in development and application. There can be little doubt, in fact, that the United States has not only relied on European sources for basic scientific research but that American engineering and industry has gained far more through basic work in engineering science imported from abroad—notably from Germany—than from the contributions of pure science in the United States. Our remarkable chemical industries have been reborn largely as the result of the confiscation of German patents during World War I. In structures, while we early originated one or two new types, most modern forms have been foreign and basic understandings have been primarily of European origin. We have, in short, been wonderful adapters and excelled in application but have made relatively few basic discoveries or contributions to basic theory.

It is obvious that World War II has put an end to our source of supply. The United States now has to rely on its own talents and resources for these fundamental understandings. To put it bluntly, we need to give far more attention than in the past to basic work in pure science—to scientific research that is not colored by the possibilities of practical application or the prospect of financial rewards. But this vital need should not lead to a neglect of the more practical aspects of the problem.

In the first place there are numerous areas of scientific understanding which are vitally important to the progress of engineering but have

never commanded the interest of researchers in pure science. Pure science, it might be noted, has its own changing areas of special interest —at the moment, for example, nuclear physics and genetics. Even problems of earlier classical physics, or botany or zoology, are, thus, neglected. Engineers must, therefore, dig out for themselves much of the basic ammunition which leads to improved practice.

We also need today far more attention to the constant improvement of our skills and abilities in application—to the development and perfection of our engineering and industrial techniques, to practical engineering science. Millions are being poured into scientific and medical research but the public viewpoint on engineering progress seems to be that, in some mysterious manner, it will muddle through on its own—if as is so often argued we give sufficient attention to providing new knowledge through research in pure science.

Thus, while many engineering techniques have been "reduced to a science," while it is essential to improvement and progress that this movement continue at an ever more rapid pace, and while it is true that we need basic research in pure science as never before in American life, we seem to be tending toward an unrealistic reliance on science alone as the solution of all our needs—a trend which has been remarkably accelerated by the propaganda of science itself. As we have attempted to show in this outline, engineering is still a practical art reinforced and strengthened, to be sure, by science stemming from both pure and engineering sources, but its major objectives are still economic and social in their nature rather than scientific, while its practice involves many elements, such as industrial and business economics, human relations and the planning and direction of labor, which require interests and abilities having little relationship to scientific teachings and for which the study of science alone does not offer an adequate preparation.

In short, if the limitations of the present American emphasis on scientific research are not properly understood, if present trends lead to a concentration of engineering education in science alone (as it seems to be doing) and if, in turn, this results (as it has in Britain) in a failure to seek ever greater understanding of, to develop and extend, the techniques of engineering and industrial application—it these errors into which we seem to be drifting are not checked and corrected, we may, in the next half century, fail dismally in carrying foward not only the promise of our past fifty years of unparalleled progress but witness the collapse and decay of man's efforts through fifty centuries of toil and struggle to build up and carry foward our Western way of life. Sir Ewart Smith's observation that it is "upon technological skill in application that the progress of industry and, consequently, the economic position of the nation will mainly depend" applies not alone to Britain but to the entire world. It's what we do with knowledge—all available knowledge—that will determine America's future strength and progress. Science is not enough.

RÉSUMÉ OF MAXWELL'S AND KIRCHHOFF'S RULES FOR NETWORK ANALYSIS

BY

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As circuit analysis is indispensable to the electrical engineers, recent treatises have been written emphasizing the use of determinants (1, 2, 3, 4)[2] matrices (5, 6) dyadics (7), and even tensors (8). For transient studies, Laplace transforms, Fourier transforms, and Heaviside's operational calculus have been well developed (9–14). However, it is always important to get the solution of a complicated network in as simple a manner as possible. The present note hopes to give a résumé of the simple rules for network analysis given by Maxwell (15) and Kirchhoff (16) as checked and extended or modified for ready use in network analysis.

Maxwell developed both the node method and the mesh method. He gave simple rules for writing down the solutions from the determinants in the node method. For the mesh method, a rule for the determinant of a 3-mesh problem was also given. Maxwell's rules have been extended to cover all branch currents in the node method (17). Kirchhoff in 1847 developed general rules for the solutions from the determinants in the mesh method. The two sets of rules will be here presented in the light of the principle of duality. The co-existence and the co-relation of these rules by two pioneers are worthy of the attention of the electrical engineers, as well the physicists and mathematicians.

I. THE NODE METHOD—MAXWELL'S RULES

Using one node potential as a reference, Maxwell developed expressions for the potential differences between any point and the reference point, say, r. The node-pairs or junction-pairs can be obtained by combining any pair of potential differences. The number of independent nodes or node-pairs is, for a simple system, equal to $n - 1$, where n is the number of nodes or points. Using admittances or admittance functions (for transient studies) instead of conductances, there is

$$i_{rs} = Y_{rs}P_{rs} = Y_{rs}\,Y_{pq}\,\frac{\Delta_{(rs,pq)}}{D}\,E_{pq}, \tag{1}$$

where

P_{rs} = the potential difference between r and s,
i_{rs} = the current in branch rs,
Y_{rs} and Y_{pq} = the admittances of branches rs and pq, and
E_{pq} = the impressed e.m.f. in branch pq.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Rule I: "The determinant D is the sum of the products of Y's taken $(n-1)$ at a time, omitting all terms which contain Y's forming closed circuits or meshes."

Rule II: "$\Delta_{(rs,pq)}$ is the sum of the products of Y's taken $(n-2)$ at a time, omitting all terms which contain Y_{rs} or Y_{pq}, and other terms either making closed circuits themselves or with the help of Y_{rs} or Y_{pq}."

Rule III: "For branches not adjacent to the e.m.f. branch, select the terms by Rule II which contain Y_{qr} (or those forming a closed circuit with Y_{qr}) and Y_{ps} (or those forming a closed circuit with Y_{ps}) as positive terms, while the corresponding terms with Y_{pr} and Y_{qs} will take a negative sign."

Rule II was formulated by the writer in 1932 (17) from Maxwell's original rule given in the Appendix of Chapter VI of ref. 15, for the branch current not adjacent to the e.m.f. branch. In the present form, all the branch currents adjacent to the e.m.f. branch can be obtained. In Rule III, we have changed the signs for the positive and negative terms as originally given by Maxwell, in order that we need not consider any minus sign attached to the determinant D.

For the current in the e.m.f. branch, Eq. 1 should be modified to the following:

$$i_{pq} = Y_{pq}(P_{pq} + E_{pq}) = Y_{pq}\frac{D'}{D}E_{pq}, \tag{2}$$

where D' is the difference of D and all the terms in D that contain Y_{pq}.

Rule IV: "D' is the sum of the products of Y's selected from D consisting of all those terms that do not contain Y of the e.m.f. branch."

If a current I_p enters the system at node-point p and leaves the system at node-point q, then

$$\frac{I_p}{P_{pq}} = \frac{D}{\Delta_{pq}}. \tag{3}$$

Rule V: "Δ_{pq} is the sum of the products of Y's taken $(n-2)$ at a time, omitting all terms containing Y_{pq} or forming a closed circuit with it, and all other terms forming closed circuits themselves."

Rules I and V are here given as stated by Maxwell. Notice that if we select from Δ_{pq} and Δ_{rs} the common terms, we can get $\Delta_{(rs,pq)}$. This is in duality to Rules II and III in the mesh method given after. Also, D' or D_{pq} is equal to the difference of D and $Y_{pq}\Delta_{pq}$. Hence we can select all the terms containing Y_{pq} and factor Y_{pq} to get Δ_{pq}. This procedure automatically satisfies the requirement of Rule V. By selecting the common terms from Δ_{pq} and Δ_{rs}, Rule III is satisfied.

Example I: Let the admittances of the different branches, Y_{12}, Y_{13}, etc. be denoted by a, b, c, d, e and f, as shown in Fig. 1. An e.m.f. E_0 will be applied in branch f. $E_0 = E_{41}$ will be considered as a voltage

rise from node 4 to node 1. The directions of the currents are as shown by the arrow-heads in Fig. 1.

From Rule I, we have

$$D = ab(c+d) + cd(a+b) + (ab+cd)e + (ad+bc)f + (ac+bd)(e+f) + (a+b+c+d)ef.$$

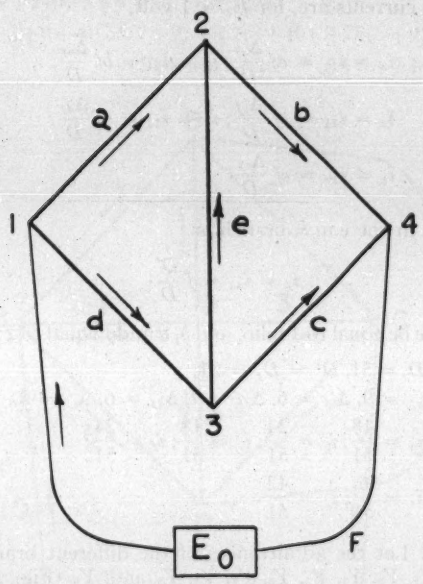

FIG. 1.

From Rule II, we have

$$\begin{aligned}\Delta_{af} &= b(c+d) + e(b+c)\\ \Delta_{bf} &= a(c+d) + e(a+d)\\ \Delta_{cf} &= d(a+b) + e(a+d)\\ \Delta_{df} &= c(a+b) + e(b+c).\end{aligned}$$

From Rule III, we have for $E_{pq} = E_{41}$, and $i_{rs} = i_{32}$,

$$\Delta_{ef} = bd - ac.$$

From Rule IV, we have

$$D' = D_f = ab(c+d+e) + cd(a+b+e) + (ac+bd)e.$$

From Rule V, we get[3]

$$\Delta' = \Delta_f = (a+b)(c+d) + (a+b+c+d)e.$$

Similarly, $\Delta_a = (b+f)(d+e) + (b+d+e+f)c.$

Notice that the common terms in Δ_f and Δ_a are bc, bd, be and ce which check with Δ_{af} obtained from Rule II. Also, $D_f = D - f\Delta_f$.

The branch currents are, for $E_0 = 1$ volt,

$$i_a = i_{12} = af\frac{\Delta_{af}}{D}; i_b = i_{24} = bf\frac{\Delta_{bf}}{D};$$

$$i_c = i_{34} = cf\frac{\Delta_{cf}}{D}; i_d = i_{13} = df\frac{\Delta_{df}}{D};$$

$$i_e = i_{32} = ef\frac{\Delta_{ef}}{D}.$$

The current in the e.m.f. branch is

$$i_f = i_{41} = f\frac{D'}{D}.$$

Let a, c and e be equal to 1 mho, and b, d and f equal to 2 mho. Then

$$D = 51, D' = D_f = 21,$$
$$\Delta_{af} = 9, \Delta_{bf} = 6, \Delta_{cf} = 9, \Delta_{df} = 6, \Delta_{ef} = 3,$$
$$i_a = \frac{18}{51}; i_b = \frac{24}{51}; i_c = \frac{18}{51}; i_d = \frac{24}{51};$$
$$i_e = \frac{6}{51}; i_f = \frac{42}{51}.$$

Example II: Let the admittances of the different branches be denoted by Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9 and Y_0 (Fig. 2). Let the values for the odd numbered branches be 1 mho, and those for the even numbered branches be 2 mho. Branch (10) will also be denoted by the subscript (0). A unit voltage E_0 will be impressed in branch (10). The directions of the currents are assumed as indicated by the arrowheads in Fig. 2.

Let node C be the reference potential node. We can write down four independent node equations according to Kirchhoff's Second Law as restated in reference 2: "The sum of the currents directed away from any node equals the sum of the currents directed towards the node." Also, we can write down the determinant form according to Maxwell. In fact, we may choose any other node as the reference node. In all cases, Rule I gives the following 125 terms for a 5-point network. We shall omit writing Y's and put down simply the subscripts.

[3] Maxwell used the symbol Δ'.

$$
\begin{aligned}
D = \; & 12[(3+6)(7+8+9) + (7+10)(8+9) + 70] \\
& + 23[(4+7)(5+8+10) + (8+9)(5+10) + 89] \\
& + 34[(1+8)(5+6+9) + (5+10)(6+9) + 50] \\
& + 14[(2+5)(6+7+10) + (6+9)(7+10) + 69] \\
& + 13(5+6)(7+8) + 24(5+8)(6+7) \\
& + (139+240)(5+6+7+8) + 1234 \\
& + (1+2+3+4)[56(7+9) + 67(8+10) + 78(5+9) + 85(6+10) \\
& \qquad + (57+68)(9+10) + (5+6+7+8)90].
\end{aligned}
$$

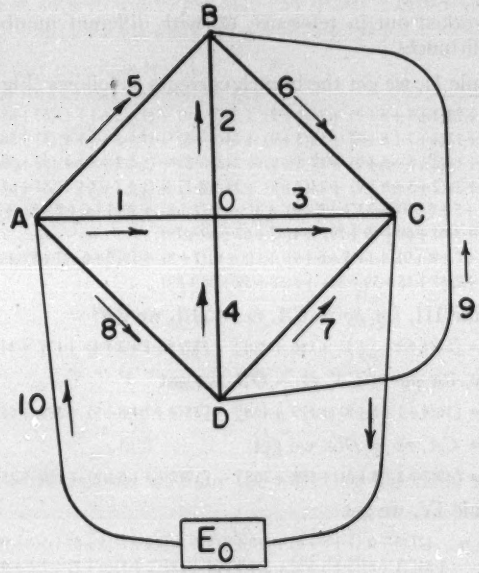

FIG. 2.

For a Wheatstone Bridge, Y_1, Y_2, Y_3, and Y_4 are zero. Choosing from D the terms common to $(1+2+3+4)$, we get

$$
\begin{aligned}
D_W = \; & 56(7+9) + 67(8+10) + 78(5+9) + 85(6+10) \\
& + (57+68)(9+10) + (5+6+7+8)90.
\end{aligned}
$$

For Y_9 and Y_0 equal to zero, we get a 5-point network with 8 branches. This was worked out by the mesh method in reference 20.

The determinant for the node method will be as follows:

$$\begin{aligned} D_P = \quad & 12[(3+6)(7+8)+78]+23[(4+7)(5+8)+58] \\ & +34[(1+8)(5+6)+56]+14[(2+5)(6+7)+67] \\ & +13(5+6)(7+8)+24(5+8)(6+7)+1234 \\ & +(1+2+3+4)[56(7+8)+78(5+6)]. \end{aligned}$$

For an Andersen Bridge, Y_1 and Y_9 will be zero, and we get

$$\begin{aligned} D_A = \quad & 23[(4+7)(5+8+10)+8(5+10)]+34[8(5+6)+6(5+10)+50] \\ & +24(5+8)(6+7)+240(5+6+7+8) \\ & +(2+3+4)[57(6+8+10)+68(5+7+10)+580+670]. \end{aligned}$$

This was worked out in reference 17, with different numbers for the branch admittances.

From Rule II, we get the branch currents as follows (Fig. 2):

$$\Delta_{50} = 146+23(4+7+8+9)+24(6+7)+34(6+9)+(1+2+3+4)(67+68+69+79)$$
$$\Delta_{60} = 345+12(4+7+8+9)+14(5+9)+24(5+8)+(1+2+3+4)(57+58+59+89)$$
$$\Delta_{70} = 238+14(2+5+8+9)+12(8+9)+24(5+8)+(1+2+3+4)(58+59+68+89)$$
$$\Delta_{80} = 127+34(2+5+8+9)+23(7+9)+24(6+7)+(1+2+3+4)(57+67+69+79)$$
$$\Delta_{10} = 34(2+5+6+9)+23(7+8+9)+3(5+6)(7+8)+39(5+6+7+8)+24(6+7)+2(67+68+69+79)+4(57+67+69+79)$$
$$\Delta_{30} = 12(4+7+8+9)+14(5+6+9)+1(5+6)(7+8)+19(5+6+7+8)+24(5+8)+2(57+58+59+89)+4(58+59+68+89).$$

From Rule III, for $pq = CA$, $rs = DB$, we get[4]

$$\Delta_{90} = [68(1+2+3+4)+146+238]-[57(1+2+3+4)+127+345].$$

Similarly, for $pq = CA$, $rs = OB$, we get

$$\Delta_{20} = [16(4+7+8+9)+179+468]-[35(4+7+8+9)+389+457].$$

For $pq = CA$, $rs = DO$, we get

$$\Delta_{40} = [38(2+5+6+9)+359+268]-[17(2+5+6+9)+169+257].$$

From Rule IV, we get

$$\begin{aligned} D' = D_0 = \quad & 12[(3+6)(7+8+9)+7(8+9)]+23[(4+7)(5+8)+5(8+9)+89] \\ & +34[(1+8)(5+6+9)+5(6+9)]+14[(2+5)(6+7)+7(6+9)+69] \\ & +13(5+6)(7+8)+24(5+8)(6+7)+139(5+6+7+8)+1234 \\ & +(1+2+3+4)[56(7+8)+78(5+6)+(5+8)(6+7)9]. \end{aligned}$$

From Rule V, we get

$$\begin{aligned} \Delta' = \Delta_0 = \quad & 12(7+8+9)+23(4+7+8+9)+34(5+6+9)+14(2+5+6+9) \\ & +24(5+6+7+8)+(1+2+3+4)[(5+6)(7+8+9)+(7+8)9]. \end{aligned}$$

For applying Rules II and III, we may proceed from D' by choosing all the terms involving the branch admittance and factoring it out, and

[4] Maxwell gave the expression for the conjugate condition between branches (14) and (23) in a 5-point network from his rule. His results check with $\Delta_{90} = 0$.

then check off all remaining terms which contain products making a closed circuit with the help of the e.m.f. branch admittance. For instance, in finding Δ_{50}, we can choose from D' all the terms involving (5) and then factoring (5) out, we get[5]

$$23(4+7+8+9)+34(1+8+6+9)+14(6+7)+13(7+8)+24(6+7)+139+(1+2+3+4)[6(7+8)+78+(6+7)9].$$

However, the remaining terms may contain products which make a closed circuit with the help of branch (10), namely, 56, 78, 13, 126, 235, 147, 348, 579 and 689. Eliminating such terms, we get

$$\Delta_{50} = 23(4+7+8+9)+34(6+9)+146+24(6+7)+(1+2+3+4)[6(7+8)+(6+7)9]$$

which checks our previous result.

Putting in numerical values, we get

$$\begin{aligned} D &= 605, \quad D' = 297, \\ \Delta_{50} &= 88, \quad \Delta_{60} = 66, \quad \Delta_{70} = 88, \quad \Delta_{80} = 66, \\ \Delta_{10} &= 77, \quad \Delta_{30} = 77, \\ \Delta_{90} &= 22, \quad \Delta_{20} = 11, \quad \Delta_{40} = 11. \end{aligned}$$

The branch currents will be

$$i_5 = i_7 = \frac{176}{605}, \qquad i_6 = i_8 = \frac{264}{605},$$

$$i_1 = i_3 = \frac{154}{605}, \qquad i_2 = i_4 = \frac{44}{605},$$

$$i_9 = \frac{44}{605}, \qquad i_0 = \frac{594}{605}.$$

As a check on the results, we have the following Kirchhoff node equations:

$$\begin{aligned} i_0 &= i_1 + i_5 + i_8 \quad \text{(at node } A\text{)} \\ i_0 &= i_3 + i_6 + i_7 \quad \text{(at node } C\text{)} \\ i_9 &= -i_2 - i_5 + i_6 \quad \text{(at node } B\text{)} \\ i_9 &= -i_4 - i_7 + i_8 \quad \text{(at node } D\text{)} \\ i_1 &= i_2 + i_3 - i_4 \quad \text{(at node } O\text{)} \end{aligned}$$

II. THE MESH METHOD—KIRCHHOFF'S RULES

If there are b branches, the number of independent meshes m is given by $(b - n + 1)$, where $(n - 1)$ is the number of independent

[5] Notice that the products making a closed circuit with the help of branch (5) have been automatically dropped out, as such products like 125 or 589 or 560 cannot appear in D or D' from Rules I and IV.

nodes. This relation was mentioned by Kirchhoff in his 1847 paper (16). The mesh method was usually attributed to Maxwell. But Kirchhoff gave simple rules for its solution two years after he announced the well-known Kirchhoff's Laws. His rules were mathematically treated by Ahrens (18) in 1897, and further discussed by Feussner (19, 20) in 1902 and 1904. We shall present these rules in the light of the principle of duality, as compared with Maxwell's rules for the node method.

Let the solution of the current i_b in mesh b in response to an e.m.f. E_a in mesh a be of the general form

$$i_b = \frac{N_{ab}}{D_Z} E_a, \tag{4}$$

where D_Z is the determinant in terms of impedances or impedance functions, Z, and N_{ab} is the co-factor of the determinant obtained by removing the a'^{th} row and the b'^{th} column of the determinant with a suitable + or − sign.

By the principle of duality, it is expected that the rule for the determinant may be formulated by noting the following considerations. In the node method, we take Y products ($n' = n - 1$) at a time, as there are n' or $n - 1$ independent nodes; in the mesh method, we take Z products m at a time, as there are m independent meshes. In the node method, we avoid meshes in the products; in the mesh method, we avoid nodes in the products. From a study of the fundamental equations and the determinants, these facts are evident. A more careful study, however, reveals the fact that while in the node method the Y products do not contain any closed loops, in the mesh method there will be no more closed loops after taking out the Z products. Hence we have, following Kirchhoff's original text,

Rule I: "D_Z is the sum of the products of Z's taken m at a time, which have the common property that, after removing elements $Z_1, Z_2 \ldots Z_m$ of such products, there leaves no closed circuit."

In the case of $m = 3$, this rule reduces to the simple rule given in Maxwell's footnote that "D is the sum of the products of the resistances taken 3 at a time, leaving out the product of any three that meet in a point."

For the response i_a to an e.m.f. E_a, the determinant form of solution will be given by

$$i_a = \frac{N_{aa}}{D_Z} E_a, \tag{5}$$

where D_Z/N_{aa} is known as the "driving-point impedance."

Rule II: "N_{aa}, the numerator, is the sum of the products of Z's taken $(m - 1)$ at a time, which have the common property that, after remov-

ing elements $Z_1, Z_2, \ldots Z_{m-1}$ of such products, there leaves only one circuit with the help of Z_a."

This was also given by Kirchhoff. Comparing with Rule V of the node method, we see that just as Y_{pq}, the admittance of the e.m.f. branch was avoided, Z_a, the impedance of the e.m.f. branch is here avoided. In the node method, all the terms that form a closed circuit with the help of Y_{pq} are omitted. In the mesh method, all the terms that meet at a node with Z_a will be omitted. In the node method, all other terms that form closed circuits themselves are omitted. So in the mesh method, there leaves no circuit formed by themselves. The duality between the two sets of rules will be even more evident as we develop Rule III for N_{ab} similar to Rule II for $\Delta_{(rs,pq)}$ in the node method.

Rule III: "N_{ab} is the sum of the products of Z's taken $(m-1)$ at a time, which have the common property that, after removing the Z elements of such products, there leaves only one circuit with the help of either Z_a or Z_b."

According to the reciprocity property of the two branches of the system, and assuming that the two mesh currents i_a and i_b coincide with the branch currents for a chosen set of m independent mesh equations, the extension from Rule II to Rule III is readily obtained.

In Rule II for the node method, we avoid all terms containing Y_{rs} and Y_{pq} and other terms making a closed circuit either by themselves or with the help of Y_{rs} or Y_{pq}. In the mesh method, we avoid similarly all terms containing Z_a or Z_b, or meeting with Z_a or Z_b at a node, and other terms such that there leaves no closed circuit formed by themselves. Just as $\Delta_{(rs,pq)}$ can be obtained from the common terms of Δ_{rs} and Δ_{pq}, N_{ab} is readily obtained from the common terms of N_{aa} and N_{bb}.

Rule IV: "For a branch current consisting of two or more mesh currents, check for sign according to the assumed directions of the mesh currents."

This is a much simpler rule than Rule III for the node method. However, in a 5-point network, we encounter mesh currents which may need the location of positive and negative terms in our solution. A similar rule to Rule III for the node method is here proposed for the mesh method by the principle of duality. We shall check this rule in Example III.

Rule V: "For a mesh current which coincides with a branch current not adjacent to the e.m.f. branch, a check of sign for the different terms is necessary. Select the terms by Rule III which contain $Z_{pr}Z_{qs}$ as positive terms, and the terms which contain $Z_{ps}Z_{qr}$ as negative terms."

As before, pq corresponds to E_{pq}, and rs corresponds to i_{rs}.

Example III: Let the impedances of the different branches be denoted by $Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_9$ and Z_0 (Fig. 3). Let the values of the Z's for the odd numbered branches be 1 ohm, and those for the even numbered branches be $\frac{1}{2}$ ohm. A unit voltage E_0 will be impressed in branch (10). As in Example II, $E_0 = E_{CA}$. The meshes are chosen such that i_5, i_6, i_7, i_8, i_9 and i_0 will serve as mesh currents as well as their own branch currents. It may be said that there are a number of sets of m independent mesh equations just as there are a number of sets of $n' = n - 1$ independent node equations. However, Max-

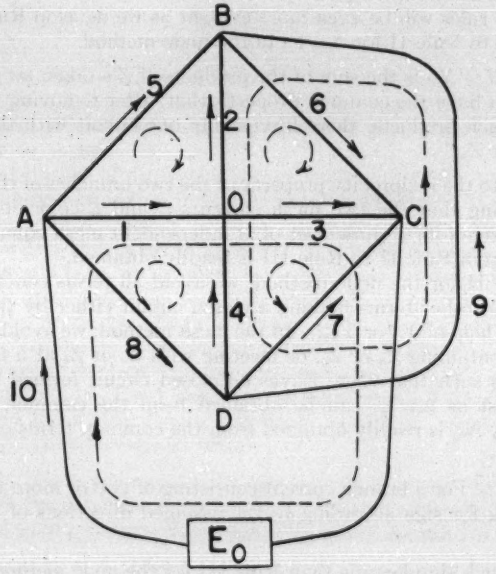

FIG. 3.

well's and Kirchhoff's Rules are given for all sets of independent node and mesh equations. We can use them for any number of sets without setting these sets of equations and determinants again and again. In this example, $m = 10 - 5 + 1 = 6$. The determinant will be of the sixth order. The saving due to Rule I is tremendous, especially if one works with Z's instead of the numercial values.

From Rule I, we get the determinant D_z composed of 125 terms. (Notice the determinant D in the node method has also 125 terms for the same problem.)

$$
\begin{aligned}
D_Z = \; & 12(3+4)[5\{(6+9)(7+8+10)+7(8+10)\} \\
& \quad +6\{(7+10)(8+9)+89\} +70(8+9) +890] \\
& +34(1+2)[7\{(8+9)(5+6+10)+5(6+10)\} \\
& \quad +8\{(5+10)(6+9)+69\} +50(6+9) +690] \\
& +127[5\{(6+9)(8+10)\} +6\{8(9+10)+90\}+890] \\
& +238[6\{(5+9)(7+10)\} +5\{7(9+10)+90\}+790] \\
& +345[7\{(8+9)(6+10)\} +8\{6(9+10)+90\}+690] \\
& +146[8\{(7+9)(5+10)\} +7\{5(9+10)+90\}+590] \\
& +13[(5+8)\{69(7+10)+790\}+589(6+7)] \\
& +24[(5+6)\{80(7+9) +790\}+560(7+8)] \\
& +167[59(8+10)+890] \\
& +278[50(6+9) +690] \\
& +358[69(7+10)+790] \\
& +456[70(8+9) +890] \\
& +567890.
\end{aligned}
$$

The combination of 10 things taken 6 at a time gives 210 terms, from which 75 terms can be taken out by the condition that Z's meeting at nodes like 1234, 1580, 2569, 3670, and 4789 should be omitted. There are 10 more terms to be omitted according to Rule I, namely, 123789, 124670, 126890, 134569, 135678, 145790, 234580, 235790, 245678, and 346890. For Z_9 and Z_0 equal to infinity, the above determinant reduces to the following:

$$
\begin{aligned}
D_Z' = \; & 12(3+4)(5+6+7+8) +34(1+2)(5+6+7+8) \\
& +127(5+6+8) +238(5+6+7) +345(6+7+8) +146(5+7+8) \\
& +13(5+8)(6+7) +24(5+6)(7+8) \\
& +167(5+8) +278(5+6) +358(6+7) +456(7+8) \\
& +5678.
\end{aligned}
$$

This checks with that given by Feussner in reference 20. D_Z' has 45 terms, while the determinant D also reduces to 45 terms in the node method for Y_9 and Y_0 equal to zero. D_Z'' for the Wheatstone Bridge may be obtained by putting Z_1, Z_2, Z_3 and Z_4 equal to infinity. Factoring either 123, 124, 341, or 342 from D_Z, we get a determinant of 16 terms,

$$
\begin{aligned}
D_Z'' = \; & 5[(6+9)(7+8+10)+7(8+10)] +6[(7+10)(8+9)+89] \\
& +70(8+9)+890
\end{aligned}
$$

or

$$
\begin{aligned}
D_Z'' = \; & 7[(8+9)(5+6+10)+5(6+10)] +8[(5+10)(6+9)+69] \\
& +50(6+9)+690.
\end{aligned}
$$

When the voltage $E_0 = E_{CA}$ is applied in branch (10), we need to

find N_{aa} or simply N_0. From Rule II, we get

$$\begin{aligned} N_0 = {} & 12(3+4)[5(6+7+9)+(6+7)(8+9)+89] \\ & +34(1+2)[7(5+8+9)+(5+8)(6+9)+69] \\ & +127[5(6+9)+6(8+9)+89]+146[8(7+9)+7(5+9)+59] \\ & +238[6(5+9)+5(7+9)+79]+345[7(8+9)+8(6+9)+69] \\ & +13[(5+8)(69+79)]+24[(5+6)(78+79+89)+56(7+8)] \\ & +167(59+89)+278(56+59+69)+358(69+79)+456(78+79+89) \\ & +56789. \end{aligned}$$

Notice that N_0 can be obtained by choosing all the products in D_Z that contain (10) and factoring (10) out. Similarly we can get N_5, N_6, N_7, N_8 and N_9. By comparing N_5 and N_0 and picking out the common terms, we get N_{50} as follows:

$$\begin{aligned} N_{50} = {} & 12(3+4)(68+78+79+89)+34(1+2)(68+78+79+89) \\ & +127(68+89)+146(78+79+89)+238(79)+138(69+79) \\ & +16789. \end{aligned}$$

This is of course the result of Rule III. While N_5 and N_0 have 75 terms each, N_{50} is simplified to 25 terms.

It may be demonstrated that although we assume certain independent meshes, we are not at all limited by such a choice. For instance, if we want to find the current in branch 1, we can form a mental picture that another set of independent mesh equations will be set up, and we are ready to solve them without writing out everything. Rule III will certainly give a direct answer for N_{10} as follows:

$$\begin{aligned} N_{10} = {} & 2(3+4)[58(6+7+9)+689]+247[5(6+9)+6(8+9)] \\ & +278[5(6+9)+69]+345[7(8+9)+8(6+9)]+358(69+79) \\ & +456(78+79+89)+56789. \end{aligned}$$

Now we shall go back to the original set of mesh equations and solve for N_{90}. By Rule III, we can write down the different terms, but we are not sure of the positive or negative signs for the different terms. Physically, we know that by the conjugate property of the two branches (9) and (10), it is quite possible to get $i_9 = 0$, and, in that case, N_{90} will be zero. Mathematically, in solving the determinant form of N_{90}, we shall be able to check the signs for the different terms. Yet, we need some supplementary rules like Rule III of Maxwell's rules for the node method. Here again, the principle of duality will be helpful. By analogy, we have formulated Rule V such that, $Z_{pr}Z_{qs}$ products will be positive and $Z_{ps}Z_{qr}$ products will be negative, where i_{rs} is to be the mesh current and E_{pq} the applied voltage. In our example, (57) products will be positive and (68) products will be negative for finding the current in branch (9), $i_{rs} = i_{DB}$, in response to a voltage $E_{pq} = E_{CA}$, in branch (10). Then

$$\begin{aligned} N_{90} = {} & 57[12(3+4)+34(1+2)+(146+238)] \\ & -68[12(3+4)+34(1+2)+(127+345)]. \end{aligned}$$

Similarly, we get the expressions for N_{20} and N_{40} as follows:

$$\begin{aligned} N_{20} = \quad & 35[(7+9)48+(4+8)79+(179+468)] \\ & -16[(7+9)48+(4+8)79+(389+457)] \\ N_{40} = \quad & 17[(6+9)25+(2+5)69+(268+359)] \\ & -38[(6+9)25+(2+5)69+(257+169)]. \end{aligned}$$

The assumed directions of i_9, i_2, and i_4 are shown in Fig. 3. Notice that at node B, i_2, i_5, i_6 and i_9 all meet at a point. In a Wheatstone Bridge, we can get i_9 from i_5 and i_6. In a 5-point network, however, we must get 3 of the 4 currents in order to specify the 4'th current, say, i_2 or i_9. Similarly, at node D, i_4, i_7, i_8 and i_9 all meet at a point. So i_9 and i_4 cannot be specified after knowing i_7 and i_8. It is evident that Rule V must be formulated so that we can handle the currents in the branches not adjacent to the e.m.f. branch. Here Maxwell's Rules are helpful to the Kirchhoff Rules.

With the numerical data given, we get

$$D_Z = \frac{605}{32}; \quad N_0 = \frac{297}{16};$$

$$N_{50} = \frac{11}{2}; \quad N_{60} = \frac{33}{4}; \quad N_{70} = \frac{11}{2}; \quad N_{80} = \frac{33}{4};$$

$$N_{90} = \frac{11}{8}; \quad N_{20} = \frac{11}{8}; \quad N_{40} = \frac{11}{8};$$

$$N_{10} = \frac{77}{16}; \quad N_{30} = \frac{77}{16}.$$

The currents will be

$$i_0 = \frac{594}{605}; \quad i_5 = i_7 = \frac{176}{605}; \quad i_6 = i_8 = \frac{264}{605};$$

$$i_9 = \frac{44}{605}; \quad i_1 = i_3 = \frac{154}{605}; \quad i_2 = i_4 = \frac{44}{605}.$$

These results check with that obtained by the node method.

As a check on the results, we have the following Maxwell mesh equations:[6]

$$\begin{aligned} (1+2+5)i_5 - 2i_6 + 2i_9 + 1i_8 - 1i_0 &= 0 \\ (2+3+6)i_6 - 2i_5 - 2i_9 + 3i_7 - 3i_0 &= 0 \\ (3+4+7)i_7 - 4i_8 + 4i_9 + 3i_6 - 3i_0 &= 0 \\ (1+4+8)i_8 - 4i_7 - 4i_9 + 1i_5 - 1i_0 &= 0 \\ (2+4+9)i_9 + 2i_5 - 2i_6 + 4i_7 - 4i_8 &= 0 \\ (1+3+10)i_0 - 1i_5 - 1i_8 - 3i_6 - 3i_7 &= E_0. \end{aligned}$$

[6] Kirchhoff's Rule I can be demonstrated similarly to Maxwell's Rule I.

The determinant D_z will be[7]

$$D_z = \begin{vmatrix} (1+2+5) & -2 & 0 & 1 & 2 & -1 \\ -2 & (2+3+6) & 3 & 0 & -2 & -3 \\ 0 & 3 & (3+4+7) & -4 & 4 & -3 \\ 1 & 0 & -4 & (1+4+8) & -4 & -1 \\ 2 & -2 & 4 & -4 & (2+4+9) & 0 \\ -1 & -3 & -3 & -1 & 0 & (1+3+10) \end{vmatrix}$$

The different mesh currents can be found by multiplying E_0 with the different cofactors, M_{05}, M_{06}, M_{07}, M_{08}, M_{09} and M_{00} and dividing by D_z.

Acknowledgment

The writer acknowledges gratefully the suggestions and criticisms of Professors E. A. Guillemin, E. W. Boehne and K. L. Wildes of Massachusetts Institute of Technology.

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[7] The determinant for a 5-point network can be simplified to the following:

$$\begin{vmatrix} (1+2+5) & (1+5) & -5 & -5 & 2 & 0 \\ 0 & (3+6) & -6 & 9 & (4+9) & 0 \\ 0 & 3 & 7 & 0 & 4 & 7 \\ 0 & 0 & (7+8) & 8 & 0 & (7+8+10) \\ 2 & 0 & 0 & 9 & (2+4+9) & 0 \\ -1 & -(1+3) & 0 & 0 & 0 & 10 \end{vmatrix}$$

There will be 20 products of third order determinants, 3 of which will be found to be zero.

INVESTIGATION OF RESISTANCE AND POWER IN EXPERIMENTAL LIGHTNING DISCHARGES

BY

HARALD NORINDER[1] AND OLOF KARSTEN[1]

INTRODUCTION

As late as about 1920 it was supposed that lightning discharges were oscillatory, and frequencies as high as half a million cycles per second were often assumed. By using an aperiodically damped antenna in conjunction with a cathode ray oscillograph, it became possible to analyze variations in the lightning strokes (1),[2] and it was discovered that the discharges were aperiodic or quasiperiodic. These results were later confirmed when lightning surges on high-tension lines were examined (2, 3). Extensive investigations of radio atmospherics (4, 5), and measurements by electromagnetic methods of the current variations in lightning paths (6), have also verified the earlier results. At some distance from the strokes, however, more or less quasiperiodic oscillations are observed, but this is adequately explained (7) by the relative importance, at different distances, of the second and third term in the equation of propagation of an electromagnetic impulse.

Before lightning variations had been experimentally analyzed, W. J. Humphreys published in this JOURNAL (8) a paper in which he discussed the discharge mechanism. He examined the behavior of an ordinary lightning path, assuming reasonable values for the inductance and capacitance of the circuit. His calculations showed that a resistance of the order of 200 ohms per kilometer in the lightning channel was enough to prevent oscillation. Obviously, then, the resistance of the channel and of its branches is of the greatest importance in any explanation of the aperiodic nature of the lightning stroke.

It has not as yet been possible to investigate the internal resistance in natural lightning paths, and one is forced to attempt the measurement of the resistance of long sparks, produced in the laboratory under conditions in which the effect of ionization from the electrodes is negligible. These sparks will presumably exhibit current variations similar to those characteristic of natural lightning. Experiments of this kind have been carried out in the Institute for High-Tension Research of the University of Uppsala, and have been described in a detailed paper (9).

[1] Institutet för Högspänningsforskning, Uppsala, Sweden.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

THE RESEARCHES AND THEIR RESULTS

The investigation was carried out by direct recording of voltage and current in a long spark gap, Figs. 1(*a*), 1(*b*), 2 and 3. In addition, a section of the spark itself was recorded by projecting it through a suitable lens system and a narrow slit, placed at right-angles to the spark

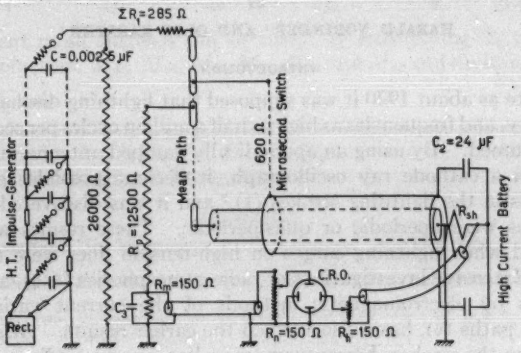

FIG. 1(*a*). Circuit diagram.*

* NOTE: The engravings for this paper are loaned through the courtesy of the Kungl. Svenska Vetenskapsakademien, Stockholm, Sweden.

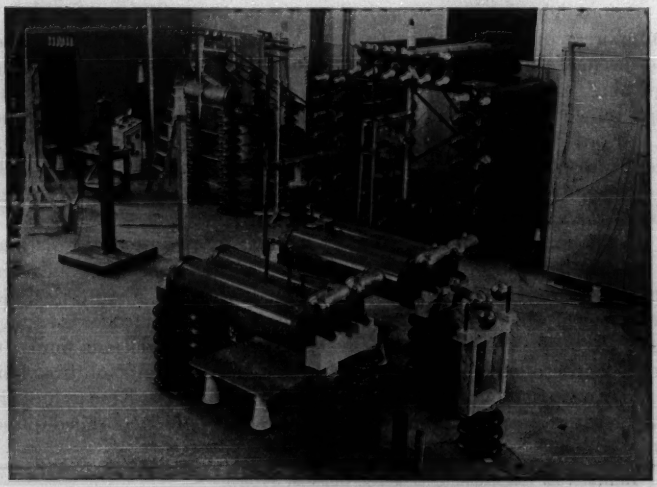

FIG. 1(*b*). Experimental arrangements of circuits.

path, on to a rotating sheet of film, Fig. 4. Various circuits were used in the experiments, with main spark lengths from 18 to 51 cm., currents from 10 to 102 kA., and durations from 1.3 to 19 μsec., the latter quantities being the interval between the commencement of the main discharge and the instant of peak current. The discharges had a maximum value of 1.2 coulomb; the maximum converted energy being 30 kWs, Fig. 5.

By the use of precision timing circuits in the oscillograph, the rate of change of current could be determined with accuracy. This enabled the inductive and resistive components of the voltage drop in the discharge path to be separated from one another, and thus the resistance

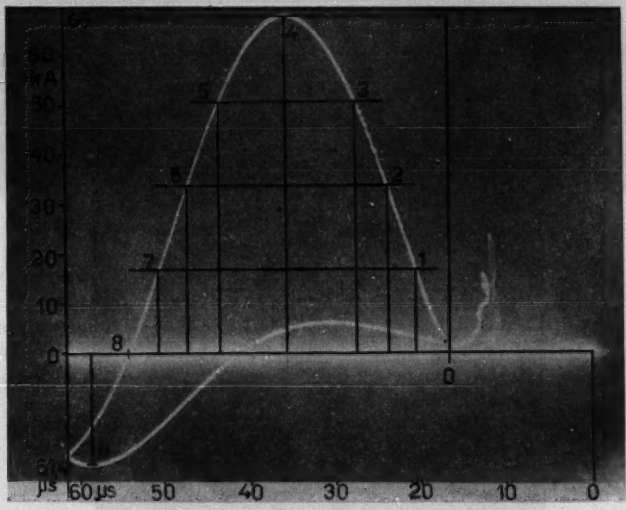

FIG. 2. Current-time oscillogram of a discharge $\frac{68\ kA}{19\ \mu s}$ i

and the power developed in the gap could be determined. The diameter of the discharge at any given instant could be measured on the photograms. The experimental method is described more fully in the paper (9) which is referred to at the end of this article. This paper also deals with special precautions which are necessary to eliminate errors due to various effects, *viz.*, the variations in inductance for different path diameters, the capacitance of the voltage measuring circuit, cathode and anode losses in the discharge path, contamination of the path by the electrodes, etc.

The conductance of the discharge path is obtained directly from the measurements as a function of the energy produced and of the quantity

discharged. Each curve in Figs. 5 and 6 is designated by a fraction whose numerator represents the peak current and whose denominator represents the time elapsed between the commencement of the main discharge and the peak current. The results thus obtained are compared with the experiments of Toepler, and Rompe and Weizel. Appreciable discrepancies in the values obtained by these investigators are established. Values obtained for the conductance of the path varied between .5 and 7 m/Ω, corresponding to an average resistance of 200Ω/km.

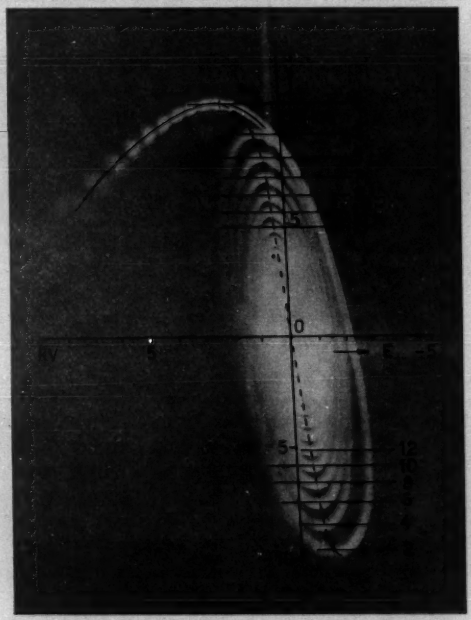

FIG. 3. Voltage-current oscillogram of a discharge $\frac{10\ kA}{4\ \mu s}$.

in the path of a lightning discharge. The power developed had a maximum value of 24 MW/cm. at 102 kA. and 8 MW/cm. at 68 kA.

The cross-sectional area of the path is determined from the photogram, and thus the energy liberated at different instants can be divided by the volume of the path, or, alternatively, by the number of particles themselves, if a permissible assumption is made concerning the spatial distribution of the particles in the arc. From the degree of molecular dissociation X_d and relative ionization X_j of the air, a relationship is

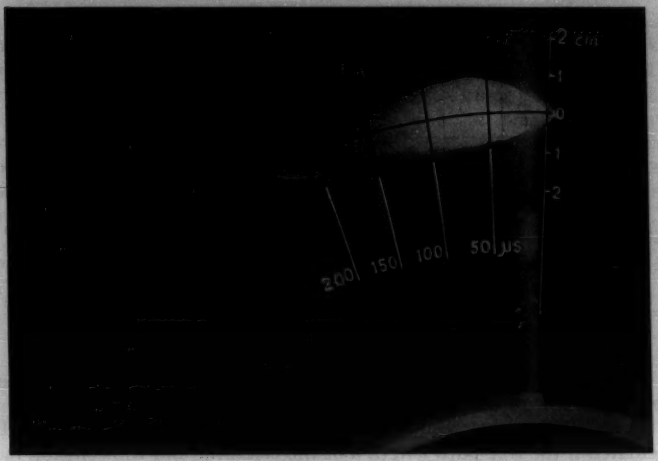

FIG. 4. Photogram of a discharge $\frac{68\ kA}{19\ \mu s}$.

derived between the internal energy and the temperature of the gas. The presence of excited atoms in the gas causes a degree of uncertainty regarding the real temperature, but this has little effect on further calculations which are intended to determine, on the one hand, the difference between the energy levels of the electrons and of the atoms, and,

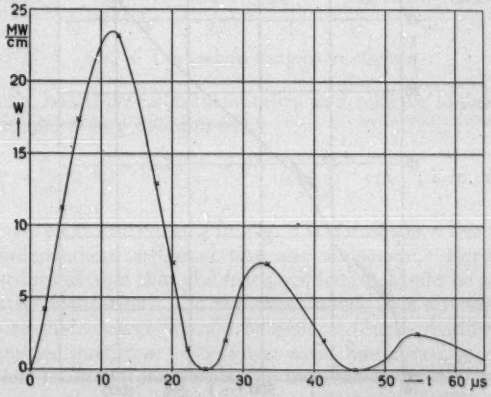

FIG. 5. Power-time diagram of a discharge $\frac{102\ kA}{12\ \mu s}$.

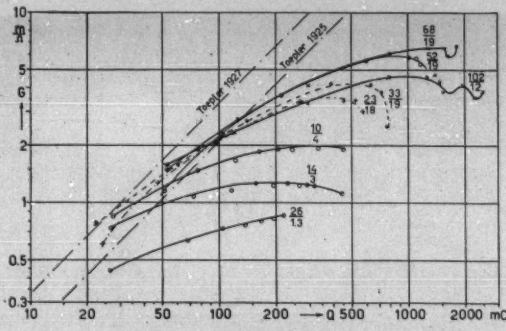

FIG. 6. Conductance plotted against electric charge.

on the other, the conductivity of the path as a function of the liberated energy per unit or of the temperature, Fig. 7.

The difference between the electron and gas temperatures can be calculated if the free-path length λ of the electrons and the collision factor η for collision between electrons and gas particles are known. These values are well-known for molecular gases, but little is known concerning a gas whose molecules are dissociated into atoms, as is the case in a lightning path. Using as a basis these data relating to a molecular gas,

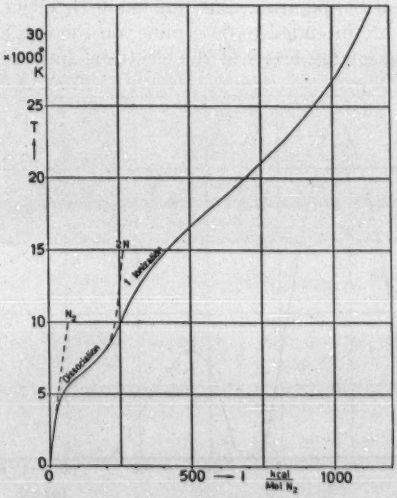

FIG. 7. Internal energy-temperature diagram.

it is possible to determine the order of magnitude of the corresponding values in the lightning path. An estimation on these lines, which takes the experimental evidence into account, gives an upper limit for the electron super-temperature; for example, 800° K. at a gas temperature of 10,000° K. and at a field strength of 1,000 V/cm. It would appear probable, however, that the actual temperature difference is appreciably lower. It is therefore apparent that, once the discharge has started, the electron super-temperature has little influence on the further development of the discharge.

It is possible, on the assumption that the temperature of the electrons is comparable with that of the gas, to calculate the conductivity

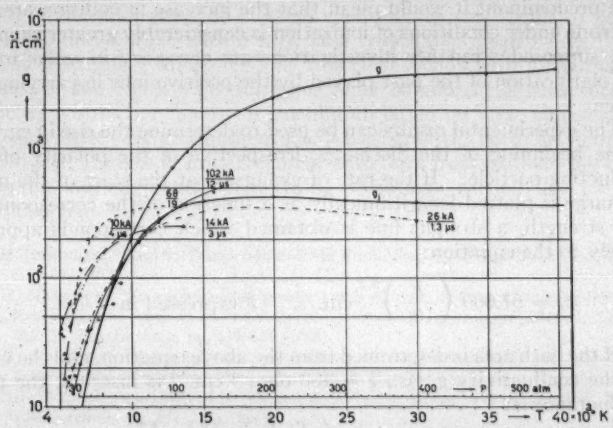

FIG. 8. Conductivity-temperature diagram.

g of the path, based on electron mobility and relative ionization. The following equation in g is obtained:

$$g = 0.96 \cdot 10^{10} \cdot \frac{X_j(1 + X_d)\lambda}{\sqrt{T_e}} \text{ in ohm}^{-1} \text{ cm.}^{-1}, \lambda \text{ in cm.}$$

The upper solid line curve g in Fig. 8 is obtained on the assumption that λ is independent of dissociation and ionization. Furthermore, it seems most improbable that the free-path length would be independent of the relative ionization. On the other hand, it is obvious that electrons, due to their charge, should be deflected more readily by ionized than by neutral particles. The lower solid line curve, g_j, in Fig. 8 is drawn in on the assumption that the increase in collision area due to ionization is ten times greater than the original area. The fine and the dotted curves with their corresponding points refer to experimental

values. As may be seen, they compare favorably with the calculated curve g_j for temperatures above 10,000° K. The conductivity is thus about 400 ohm^{-1} cm.$^{-1}$, and is fairly independent of temperature between 10,000 and 30,000° K.

The conclusions are obtained on the assumption that the current is carried principally by the electrons. The authors of the tenth reference state that under conditions of high ionization the current is carried to a considerable extent by the positive ions. The investigation described in this article has not provided any concrete evidence on which to postulate the polarity of the particles carrying current. Nevertheless, it appears that particles of one polarity predominate. If positive ions were predominant it would mean that the increase in collision area for electrons under conditions of ionization is considerably greater than has been supposed. Further investigations are necessary in order to obtain clarification of the part played by the positive ions in carrying the current.

The experimental results can be used to determine the rise in current at the beginning of the discharge, irrespective of the polarity of the conducting particle. If the rate of expansion at the start of the main discharge is plotted logarithmically as a function of the corresponding field strength, a straight line is obtained which corresponds approximately to the equation:

$$v_z = 67{,}000 \left(\frac{F}{100}\right)^{0.3} \text{ cm./s.} \quad (F \text{ expressed in V/cm.})$$

If the path area is determined from the above equation, and the value for the conductivity g, *viz.*, $g = 400$ ohm^{-1} cm.$^{-1}$ is inserted, the total conductance is:

$$G = 5.64 \cdot 10^{12} \cdot \left(\int_0^t \left(\frac{F}{100}\right)^{0.3} dt\right)^2.$$

If the latter equation is combined with the values for the inductance and capacitance of the lightning path, the rate of increase of current during a lightning stroke to earth may be calculated in accordance with Maxwell's equations.

This, however, involves certain additional problems, clarification of which first will be necessary.

CONCLUSIONS

The resistance and the power in artificial lightning paths with lengths up to 51 cm. have been investigated by cathode ray oscillographs recording simultaneously voltages and currents. The artificial lightning paths were long enough to prevent the influence of ionization from the electrodes on the result. Currents up to 102 kiloamperes, total charges of 1.2 coulombs and a total quantity of energy of 30 kilowattseconds

were used in the circuits. The increase of the spark diameter with time was measured on a high-speed rotating film with a sensitive lens system.

The experimental data were used as the basis for calculation and discussion of ionization conditions within the spark. The conductance of the path of heavy lightning strokes was found to be proportional to the cross section of the path, and the conductivity was about $400\Omega^{-1}$/cm.$^{-1}$

The main purpose of the investigation was to measure the resistance within long sparks carrying current of the same order as lightning strokes occurring in the open atmosphere. These experimental results were applied to natural lightning discharges. Assuming plausible values of the diameter, the channel conductance was found to lie between 0.5 and 7 m/Ω, with 5 m/Ω as a reasonable mean value. In a vertical lightning 1 kilometer long this corresponds with a channel resistance of 200 ohms. To this value must be added the resistances of branches within the cloud. A calculation based on these values shows that only aperiodic or quasiperiodic variations will occur in lightning discharges. This is in full agreement with data from cathode-ray oscillographic measurements of lightning variations carried out twenty-five years ago.

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Helicopter Industry Awarded Collier Trophy.—The youthful helicopter industry recently acknowledged with "a deep feeling of pride" the award of the coveted Collier Trophy, one of aviation's highest tributes, to the "Helicopter Industry, the Military Services and the Coast Guard for development and use of rotary-wing aircraft for air rescue operations."

This expression was made by Harvey Gaylord, Vice-President of Bell Aircraft Corporation and Chairman of the Helicopter Council, Aircraft Industries Association of America.

"The Helicopter Industry is barely twelve years old and it is thus both gratifying and inspirational that rotary-wing aircraft have served so well in saving the lives of almost 12,000 of our fighting men," Mr. Gaylord said. "It is with a deep feeling of pride that the Industry shares the Collier Trophy with the Military Services and the Coast Guard. The full utilization of the helicopter in Korea could not have been possible without the pioneering efforts in previous years by the Military Services and the Coast Guard.

"Helicopters, in the hands of capable and courageious pilots, have provided the world's fastest and most effective method of evacuating wounded or stranded fighting men."

Concluding the statement in behalf of the Industry, Mr. Gaylord emphasized "We are hopefully anticipating and preparing for the day when peace will permit civil production to be resumed and our economy benefitted by the full utilization of the helicopter in the service of the public."

Newly-developed crash fire trucks on order for the Air Force will get there first with the most fire-fighting equipment, can "stand-up" to hot aircraft fires with operating personnel completely enclosed and protected by insulated and ventilated compartments. Lighter construction and more powerful motors enable trucks to hit 60 mph. in 61 seconds, maintain top speed of 70 plus. Cabs are forward to the front bumper, contain joystick controls for three bumper type fog foam nozzles of 35 gpm. and a cab roof mounted turret nozzle of 200 gpm.

A light weight gyro compass which will withstand rough treatment in any type of vehicle has been developed under contract with the Army Corps of Engineers, offers precise navigation in any area and under all conditions. The self-contained, self-sufficient unit weighs only 67 lbs., as compared to the 550 lb. World War II model. Designed to operate in temperatures ranging from −65 to plus 130° F., the compass is protected from water and dust; the sensitive element is suspended in a high density fluid which offers excellent protection. Built by Arma Corp. of Brooklyn, the compass will soon undergo field evaluation, will be standardized within two years.

Standardization of engines and parts is paying off for the Army Ordnance Corps, which states that in contrast to World War II practice of using six to ten different engines for the same tank, the new deal in design permits four engines to be used in eleven different vehicles. Sixty per cent of the 2000 parts used in these four engines are interchangeable, while none of the 5000 parts used in World War II engines were interchangeable. In the Walker Bulldog family of eleven vehicles, over 60 major components, including engines, generators, ignition, and suspension systems are interchangeable.

THE USE OF THE NULL-UNIT FUNCTION IN GENERALIZED INTEGRATION

BY

J. J. SMITH[1] AND P. L. ALGER[1]

1. INTRODUCTION

A history of generalized integration with numerous references has been compiled by Davis (1).[2] He gives the various definitions of generalized integration that have been proposed by Liouville, Cauchy, Fourier, Abel, Riemann, Heaviside, Bromwich, Carson and others. As one would expect with so many different definitions, troubles arise because they do not always give the same results. Davis discusses these differences and suggests reasons for them.

To the authors the cause of these differences seems to lie deeper than some of the explanations covering details given by Davis. For instance, he refers to Cayley's discussion of Riemann's work on generalized integration as follows: "Riemann approaching the subject from Taylor's series found himself inextricably tangled in difficulties with the complementary function" and again "What is the real meaning of a complementary function containing an infinity of arbitrary constants?"

These difficulties with the complementary functions are encountered in the case of ordinary integration where if a function $f(x)$ is first integrated from 0 to x and the result then differentiated we obtain $f(x)$, on the other hand if we first differentiate it and then integrate from 0 to x we obtain $f(x) - f(0)$. Further, if one were foolhardy enough to try to perform a single differentiation by differentiating n times and then integrating $n - 1$ times, $n - 1$ constants of integration would be obtained.

In a recent paper (2) the authors showed how the introduction of the Null-Unit function $H(x, x_0)$ which is equal to 0 if $x < x_0$ and is equal to 1 if $x > x_0$ played an important part in eliminating the occurrence of unknown constants in connection with ordinary integration and differentiation and also differential equations and in making such operations commutative. The method of Schwartz (3) was used to handle the discontinuities introduced by $H(x, x_0)$.

It seems almost essential that two successive integrations of half order, for instance, give the same result as ordinary integration of one order if the chaotic situation encountered by Riemann is to be avoided. It is therefore of interest to see whether the introduction of the Null-Unit function into generalized integration will give a consistent system which avoids the difficulties encountered when the Null-Unit function is not used. Thus in this paper we shall consider principally operations on a class of functions which are zero when $x < x_0$; which is accomplished by multiplying any given function by the Null-Unit function $H(x, x_0)$.

[1] General Electric Company, Schenectady, N. Y.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

It is found possible to develop a consistent theory for this class of functions. In so doing we avoid the difficulties encountered where the Null-Unit function is not used in generalized integration. This has the advantage that it does not disturb any of the existing methods for handling continuous functions. It simply provides a more direct and convenient method of solving problems with functions which are zero when $x < x_0$. It will be necessary to give some of the results in the form of their representation in mass space as developed by Schwartz since the functions given by this representation do not exist in a mathematical sense.

In this paper it is shown that (1) starting with a definition proposed by Liouville for the n^{th} order of integration of $f(x)$, (2) modifying this to limit the integration to a given interval x_0 to x, and (3) finally introducing the Null-Unit function as a multiplier, leads to a successful definition of the generalized integration process that is fully commutative with generalized differentiation.

With these modifications it can be shown that the functions used in the definitions of Abel, Riemann, Heaviside, Bromwich and Carson are actually of the type $f(x)\ H(x, x_0)$. The method of Liouville, although employing the same integral as Carson, applied to x^{-m} gives a different result from the correct one obtained by Carson. It can be shown that Liouville's method actually gives the same result as Carson's method when the Null-Unit function implicit in his equations is not neglected, and that Liouville's incorrect result is due to his neglect of this Null-Unit function. These other definitions, together with their relation to the definitions in this paper, will be discussed in a later paper. The Liouville definition is discussed in section 6 of this paper.

V. 19

2. DEFINITION OF GENERALIZED INTEGRATION AND DIFFERENTIATION

The usual definition of a differential coefficient is

$$\frac{d}{dx}f(x) = \operatorname*{Lim}_{h\to 0}\frac{f(x+h) - f(x)}{h}, \tag{1}$$

which is called the differential coefficient on the right. A similar definition for the differential coefficient on the left is

$$\frac{d}{dx}f(x) = \operatorname*{Lim}_{h\to 0}\frac{f(x) - f(x-h)}{h}. \tag{2}$$

In generalized integration from x_0 to x where $x_0 < x$ it will be found necessary for reasons which appear below to start with the latter definition.

The second differential coefficient may be written

$$\frac{d^2}{dx^2}f(x) = \operatorname*{Lim}_{h\to 0}\frac{f(x) - 2f(x-h) + f(x-2h)}{h^2}. \tag{3}$$

Continuing this process a generalized expression for the n^{th} differential coefficient is found to be (4)

$$\frac{d^n}{dx^n}f(x) = \operatorname*{Lim}_{h\to 0} \frac{f(x) - nf(x-h) + \dfrac{n(n-1)}{2!}f(x-2h) + \cdots}{h^n}. \quad (4)$$

If n is a positive integer, the numerator of Eq. 4 is a closed series of $n+1$ terms. For example, if $f(x) = x^m$ and $n = 2$, we have from Eq. 4:

$$\frac{d^2x^m}{dx^2} = \operatorname*{Lim}_{h\to 0} \frac{x^m - 2(x-h)^m + (x-2h)^m}{h^2}, \quad (5)$$

which, if h is small, becomes:

$$\frac{d^2x^m}{dx^2} = m(m-1)\,x^{m-2}, \quad (6)$$

a familiar result.

If n is negative, or fractional, however, the numerator of Eq. 4 is an infinite series and the result becomes indeterminate. For example, if the values of $(x - rh)^m$ are each expanded into a series:

$$(x - rh)^m = x^m - mrhx^{m-1} + \cdots, \quad (7)$$

Eq. 4 becomes:

$$\frac{d^nx^m}{dx^n} = \operatorname*{Lim}_{h\to 0} \frac{x^m(1-1)^n + hnmx^{m-1}(1-1)^{n-1} + \cdots}{h^n}, \quad (8)$$

which is indeterminate.

The usual definition of a definite integral is

$$\int_{x_0}^{x} f(x)dx = \operatorname*{Lim}_{h\to 0,\ rh\to x-x_0} [f(x) + f(x-h) + f(x-2h) + \cdots + f(x-rh)]\,h \quad (9)$$

and as $x_0 \to -\infty$ this might be written

$$\int_{-\infty}^{x} f(x)dx = \operatorname*{Lim}_{h\to 0} [f(x) + f(x-h) + f(x-2h) + \cdots]\,h. \quad (10)$$

By comparison we might take Eq. 4 to correspond to the definition of n^{th} order integration between the limits $-\infty$ and x, writing

$$\int_{-\infty}^{x} f(x)dx^n = \operatorname*{Lim}_{h\to 0} \left[f(x) - \binom{-n}{1} f(x-h) + \binom{-n}{2} f(x-2h) + \cdots \right] h^n. \quad (11)$$

It is seen that Eq. 11 is obtained directly from Eq. 4 by substituting

$-n$ for n. It, therefore, appears that the limits for integration (negative differentiation) implied in Eq. 4 are between x and $-\infty$. This expression can only be interpreted, in cases where $f(x) \to 0$ of a high enough order when $x \to -\infty$, as already seen in Eq. 8.

Similarly, when a start is made from Eq. 1 with the sign reversed, we obtain the comparable formula

$$\int_x^{\infty} f(x)dx^n = \operatorname*{Lim}_{h\to 0}\left[f(x) - \binom{-n}{1} f(x+h) + \binom{-n}{2} f(x+2h) + \cdots\right] h^n, \quad (12)$$

which again gives indeterminate results unless $f(x) \to 0$ of a high enough order when $x \to \infty$.

It should be noted this gives the opposite sign for the differential coefficient to that given by Eq. 11. This will cause no confusion, however, if the limits are also used in connection with differentiation.

By a slight modification of the above we can make the interval of integration finite as follows. Define the n^{th} generalized integral[3] of $f(x)$ over the interval x_0 to x where $x > x_0$ as

$$\int_{x_0}^{x} f(x)dx^n = \operatorname*{Lim}_{h\to 0} h^n\left[f(x) - \binom{-n}{1} f(x-h) + \binom{-n}{2} f(x-2h) + \cdots + (-1)^r \binom{-n}{r} f(x-rh)\right] \quad (13)$$

where as before the index n on dx^n shows the order of the integration and $rh = x - x_0$ the interval over which the function is integrated. If $x_0 > x$ this should be modified as in the case of the integral from x to ∞ above to obtain

$$\int_x^{x_0} f(x)dx^n = \operatorname*{Lim}_{h\to 0} h^n\left[f(x) - \binom{-n}{1} f(x+h) + \binom{-n}{2} f(x+2h) + \cdots + (-1)^r \binom{-n}{r} f(x+rh)\right]. \quad (14)$$

Care should be taken to note with these definitions that in general

$$\int_{x_0}^{x} f(x)dx^n \neq -\int_x^{x_0} f(x)dx^n, \quad (15)$$

which is commonly used in connection with a single integration for which it still holds. The reason is obvious when the two formulas as defined above are compared.

If $n < 0$ the same formula gives a definition for generalized differentiation of order $-n$ which may be denoted by $\int_{x_0}^{x} f(x)dx^{-n}$ or $D_{x_0}{}^{x} f(x)dx^{-n}$.

[3] In the following we shall use the symbol $\binom{n}{r}$ to denote the coefficients of a binomial expansion. This notation has been used by others and is convenient.

Davis uses the symbol ${}_{x_0}D_x^n f(x)$ which is perhaps simpler but does not denote the independent variable involved in the differentiation which is important in functions containing several independent variables. However not much use will be made of the direct formula for generalized differentiation since in many cases it leads to impulse functions and these are more readily handled by the method of Schwartz using functional representation as will be seen later in the paper.

C. COMMUTABILITY OF ITERATIVE OPERATIONS IN GENERALIZED INTEGRATION

In a previous paper by the authors, considerable emphasis was placed on the necessity for having iterative integral and differential operations reversible in applications to physical problems. This was accomplished by introducing the Null-Unit function $H(x, x_0)$. Since the definitions of generalized integration and differentiation given above coincide reasonably well with the usual definitions for whole integrations and differentiations we may suspect that operations with these definitions will not be in general reversible. That we are correct in this suspicion may be seen as follows.

In the following we use $\int_{x_0}^{x} f(x)dx^{-n}$ corresponding to differentiation. It is simpler to write, as it eliminates a large number of terms involving $-n$ on the right-hand side which would occur if we used $\int_{x_0}^{x} f(x)dx^{n}$. We have

$$\int_{x_0}^{x} f(x)dx^{-n} = \operatorname*{Lim}_{h\to 0} h^{-n}\left[f(x) - \binom{n}{1} f(x-h) + \binom{n}{2} f(x-2h) + \cdots + (-1)^r f(x-rh)\right] \tag{16}$$

and

$$\int_{x_0}^{x} dx^{-m} \int_{x_0}^{x} f(x)dx^{-n} = \operatorname*{Lim}_{h\to 0} h^{-m-n}\left[\begin{array}{l} f(x) - \binom{m}{1} f(x-h) + \binom{m}{2} f(x-2h) + \cdots + (-1)^r \binom{m}{r} f(x-rh) \\ \quad - \binom{n}{1} f(x-h) + \binom{n}{1}\binom{m}{1} f(x-2h) + \cdots - (-1)^r \binom{n}{1}\binom{m}{r-1} f(x-rh) + (-1)^{r+1}\binom{n}{1}\binom{m}{r} f(x-(r+1)h) \\ \qquad + \binom{n}{2} f(x-2h) + \cdots + (-1)^r \binom{n}{2}\binom{m}{r-2} f(x-rh) + \cdots + (-1)^{r+2}\binom{n}{2}\binom{m}{r} f(x-(r-2)h) \\ \qquad\quad + \cdots \qquad + \cdots \end{array}\right]. \tag{17}$$

Now

$$(1-y)^n = 1 - \binom{n}{1} y + \binom{n}{2} y^2 - \cdots + (-1)^r \binom{n}{r} y^r + \cdots \tag{18}$$

and

$$(1-y)^m = 1 - \binom{m}{1} y + \binom{m}{2} y^2 - \cdots + (-1)^r \binom{m}{r} y^r + \cdots \tag{19}$$

and it is evident that the coefficient of $f(x - ph)$ where $p < r$ in the [] in Eq. 17 is the coefficient of y^p in the expansion $(1 - y)^{m+n}$ which is $(-1)^p \binom{m+n}{p}$ and thus

$$\int_{x_0}^{x} dx^{-m} \int_{x_0}^{x} f(x)dx^{-n}$$

$$= \lim_{h \to 0} h^{-m-n} \left[\begin{array}{l} f(x) - \binom{m+n}{1} f(x-h) + \cdots + (-1)^r \binom{m+n}{r} f(x-rh) \\ \quad + (-1)^{r+1} \binom{n}{1} \binom{m}{r} f(x-(r+1)h) \\ \quad + (-1)^{r+1} \binom{n}{2} \binom{m}{r-1} f(x-(r+1)h) + (-1)^{r+2} \binom{n}{2} \binom{m}{r} f(x-(r+2)h) \\ \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \quad + (-1)^{r+1} \binom{n}{r-1} \binom{m}{1} f(x-(r+1)h) + \cdots (-1)^{2r+1} \binom{n}{r-1} \binom{m}{r} f(x-(2r+1)h) \end{array} \right]. \quad (20)$$

Since $x - x_0 = \lim_{h \to 0} rh$, the sum of the first $(r + 1)$ terms on the right-hand side of this equation is by definition $\int_{x_0}^{x} f(x)dx^{-m-n}$ and since the sum of the remaining terms is not zero in general (we shall not attempt to sum them since this does not seem to be a simple matter) it is evident that with the definitions we have given so far

$$\int_{x_0}^{x} dx^{-m} \int_{x_0}^{x} f(x)dx^{-n} \neq \int_{x_0}^{x} f(x)dx^{-m-n}. \quad (21)$$

This confirms our expectation that with the proposed definitions in Eqs. 13 and 14 integration and differentiation are not commutative. However, an examination will show that the difference between the consecutive operations in the left-hand side of Eq. 20 and the single operation on the right-hand side involves only terms of the form $f(x - (r + k)h)$ so that all these terms are of the form $f(x_0 - kh)$ where $k > 0$. In other words, they involve the value of $f(x)$ at points outside the region of integration or differentiation which is from x_0 to x.

This situation makes us realize that our definition is not complete enough when we have to consider such iterative operations. We have the option of including values of $f(x)$ outside the interval of integration as in Eq. 20 and then we arrive at Eq. 21. On the other hand we might agree that in iterative operations we will not permit the introduction of any values of $f(x)$ outside the range of integration and with this understanding the result in Eq. 21 may easily be modified so that the operations are reversible. This can be done by letting these definitions stand and using instead of $f(x)$ the function $f(x)\ H(x, x_0)$ the value of which is zero when $x < x_0$ due to the definition of $H(x, x_0)$.

If this is done then the terms in the lines 2, $3 \cdots r$ in the square

bracket in Eq. 20 become

$$\begin{aligned} &f(x-(r+1)h)H(x_0-h, x_0)\\ &f(x-(r+2)h)H(x_0-2h, x_0)\\ &\cdots\cdots\cdots\cdots\cdots\cdots\\ &f(x-(2r+1)h)H(x_0-(r+1)h, x_0) \end{aligned} \tag{22}$$

which are all zero due to the presence of the Null-Unit function which is zero in each case since $x_0 - nh < x_0$. Thus

$$\int_{x_0}^{x} dx^m \int_{x_0}^{x} f(x)H(x, x_0)dx^n = \int_{x_0}^{x} f(x)H(x, x_0)dx^{m+n}. \tag{23}$$

It is interesting to try the simple case of a single differentiation followed by a single integration on $f(x)$ in Eq. 17. Here the square bracket consists of only two lines which cancel term by term except for $f(x) - f(x-(r+1)h)$. Thus the final result is $f(x)H(x, x_0)$ when the Null-Unit function is used and is the customary one $f(x) - f(x_0)$ with the constant of integration $f(x_0)$ when the Null-Unit function is not used.

Another case of reversibility of operations which may be considered is where $f(x) \to 0$ of a high enough order, when $x \to +\infty$ or $-\infty$, so that the sum of the terms of the type $f(x-(r+p)h)$ on lines $2, 3, \cdots r$ in Eq. 20 tends to zero. In such a case we may write from Eq. 12

$$\int_{x}^{\infty} dx^m \int_{x}^{\infty} f(x)dx^n = \int_{x}^{\infty} f(x)dx^{m+n}, \tag{24}$$

where the sum of the terms of the type $f(x-(r+p)h) \to 0$ as $x \to \infty$ or similarly from Eq. 11

$$\int_{-\infty}^{x} dx^m \int_{-\infty}^{x} f(x)dx^n = \int_{-\infty}^{x} f(x)dx^{m+n}, \tag{25}$$

where the sum of the terms of the type $f(-x-(r+p)h) \to 0$ as $x \to -\infty$.

4. EXAMPLES OF GENERALIZED INTEGRATION

In the following the discussion will be restricted to functions of one of the above two types: (1) generalized integration over the interval (x, x_0) of the function $f(x)H(x, x_0)$, (2) generalized integration over the interval (x, ∞) or the interval $(x, -\infty)$ of the function $f(x)$ which (under the conditions referred to above) tends to zero of a high enough order as x tends to $+\infty$ or $-\infty$, respectively:

(1) *As an example involving a function of the type* $\mathrm{f(x)H(x, x_0)}$ *let* $\mathrm{f(x) = 1}$. *Then*

$$\int_{x_0}^{x} H(x, x_0)dx^n = \operatorname*{Lim}_{h\to 0}\left[1 - \binom{-n}{1} + \binom{-n}{2} + \cdots + (-1)^r \binom{-n}{r}\right] h^n H(x, x_0), \tag{26}$$

where $rh = x - x_0$. Now

$$(1-y)^{-n} = 1 - \binom{-n}{1}y + \binom{-n}{2}y^2 + \cdots + (-1)^r\binom{-n}{r}y^r + \cdots \quad (27)$$

$$(1-y)^{-1} = 1 + y + y^2 + \cdots + y^r + \cdots \quad (28)$$

and hence the quantity in the [] in Eq. 26 is equal to the coefficient of y^r in the expansion of $(1-y)^{-n-1}$ which is

$$(-1)^r\binom{-n-1}{r} = \frac{(n+1)(n+2)\cdots(n+r)}{r!} \quad (29)$$

and thus[4]

$$\int_{x_0}^{x} H(x, x_0)dx^n = \lim_{h\to 0} \frac{(n+1)(n+2)\cdots(n+r)}{r!\,r^n} r^n h^n H(x, x_0)$$

$$= \frac{(x-x_0)^n H(x, x_0)}{\Pi(n)}. \quad (30)$$

This result is the same as that obtained by usual methods when n is a positive integer. It does not formally agree with that obtained by the usual method when n is a negative integer corresponding to ordinary differentiation, which gives a value zero. However since when n is negative the symbolic expression $(x-x_0)^n H(x, x_0)/\Pi(n)$ involves infinite values when $x \to x_0$ the difficulty of handling these infinite values in function space will be avoided by using the method of Schwartz and studying their functional representation in mass space as shown later. For this reason we restrict for the present the use of Eq. 30 to values of $n > 0$ although it can be shown to hold when $n > -1$.

When the function $f(x)H(x, x_0)$ corresponds to $(x-x_0)^n H(x, x_0)/\Pi(n)$ its q^{th} generalized integral can be found by making use of the fact that iterative operations are commutative. From Eq. 30 write

$$\int_{x_0}^{x} H(x, x_0)dx^p = \frac{(x-x_0)^p H(x, x_0)}{\Pi(p)}, \quad (31)$$

also

$$\int_{x_0}^{x} H(x, x_0)dx^{p+q} = \frac{(x-x_0)^{p+q} H(x, x_0)}{\Pi(p+q)}. \quad (32)$$

Hence from Eq. 23

$$\int_{x_0}^{x} \frac{(x-x_0)^p H(x, x_0)dx^q}{\Pi(p)} = \frac{(x-x_0)^{p+q} H(x, x_0)}{\Pi(p+q)} \quad (33)$$

[4] In the remainder of this paper we shall use $\Pi(n) = \Gamma(n+1)$ since it makes all the formulas more symmetrical.

and this is the general formula required in most physical applications. It can be derived by many of the other formulas for generalized integration such as those of Abel, Bromwich, Carson, Heaviside which will be discussed in a later paper. By most authors it is written without the $H(x, x_0)$ which, however, is an essential part of the formula if iterative operations are to be commutative.

It will be noted that in the above x and x_0 appear (1) in the integral limits, (2) in the function to be integrated $(x - x_0)$, and (3) in the Null-Unit function $H(x, x_0)$. If this relationship does not hold, the formula becomes much more complicated. For instance let it be required to determine

$$\int_{x_0}^{x} x^p H(x, x_0) dx^q. \tag{34}$$

The first question to be asked is whether $0 > x_0$ or $0 < x_0$.

If $0 < x_0$ then the application of Eq. 30 will give us the integral

$$\int_{0}^{x} x^p H(x, 0) dx^q \tag{35}$$

and values of $x < 0$ cannot be included with this type of integral if it is wished to retain the property that iterative operations are commutative. If $0 < x_0$ then put the integral in the form

$$\int_{x_0}^{x} (x - x_0 + x_0)^p H(x, x_0) dx^q.$$

$(x - x_0 + x_0)^p$ can then be expanded in ascending or descending powers of $x - x_0$, whichever is valid, and Eq. 30 can be used to give the result in the form of a series.

These formulas do not always follow the simple rules we are accustomed to in connection with ordinary integration as would be expected from their more complicated definitions. For instance we may derive an expression for the general case similar to the ordinary integral of x^n as follows. Put x_0 equal to zero in Eq. 30. Thus

$$\left[\int_0^x dx^m - \int_0^{x_1} dx^m\right] \frac{x^n}{\Pi(n)} H(x, 0) = \left[\frac{x^{n+m}}{\Pi(n+m)} - \frac{x_1^{n+m}}{\Pi(n+m)}\right] H(x, 0). \tag{36}$$

In the special case of $m = 1$ corresponding to ordinary integration the integrals on the left-hand side may be combined and written $\int_{x_1}^{x} dx$, giving the well-known expression. However this combination of the integrals obviously cannot be made in the general case.

By a slight modification of the above when $n - m$ is negative using

the method of Schwartz as in the next section to take care of the impulse functions which would otherwise enter into the work, it can be shown that

$$\left[\int_0^{x_1} dx^{-m} - \int_0^{x} dx^{-m}\right]\frac{x^{-n}}{\Pi(-n)} H(x, 0) = \left[\frac{x_1^{-n-m}}{\Pi(-n-m)} - \frac{x^{-n-m}}{\Pi(-n-m)}\right] H(x, 0). \quad (37)$$

If x_1 is taken equal to ∞ the term in x_1 on the right-hand side becomes equal to zero. Then in the case of a single integration the integrations on the left-hand side may be combined to give a single integral equivalent to $\int_x^\infty dx$ which is the well-known ordinary result. However in the general case the integrals cannot be combined and the integral from 0 to ∞ on the left-hand side must be retained as its deletion may cause impulse functions to enter the picture as will be found when we consider the method of Schwartz.

It may also be interesting to note that if $x_0 > x$ from Eq. 14

$$\int_x^{x_0} \frac{(x_0 - x)^n H(x_0, x) dx^m}{\Pi(n)} = \frac{(x_0 - x)^{n+m} H(x_0, x)}{\Pi(n+m)}. \quad (38)$$

and the limit of this formula as $x_0 \to \infty$ does not exist in the usual case. This is typical of the ambiguous results generally obtained when an attempt is made to get rid of the Null-Unit function $H(x_0, x)$ by putting the step due to it at $x_0 \to \infty$.

(2) *The following shows examples of generalized integration with the limits* x *and* $+\infty$ *or* $-\infty$. *For instance from Eq. 11*

$$\int_{-\infty}^{x} e^x dx^n = \operatorname*{Lim}_{h \to 0} h^n \left[e^x + ne^{x-h} + \frac{n(n+1)}{2!} e^{x-2h} + \cdots\right]. \quad (39)$$

This result was derived by Liouville and used by him as a basis for the generalized integration of other functions by expressing them in forms involving exponentials. Liouville did not state the limits of integration but simply wrote

$$d^n e^x / dx^n = e^x, \quad (40)$$

which is more fully written as in Eq. 39. By analogy he wrote

$$d^n e^{-x} / dx^n = (-1)^n e^{-x}. \quad (41)$$

This latter formula is hardly justifiable if the limits $-\infty$ and x are implied since $e^{-x} \to \infty$ as $x \to -\infty$. It also leads to confusion since in some cases only the real part of $(-1)^n$ or $\cos n\pi$ is taken.

If Eq. 12 is used with e^{-x} we obtain

$$\int_x^\infty e^{-x}dx^n = \operatorname*{Lim}_{h\to 0} h^n\left[e^{-x} + ne^{-x-h} + \frac{n(n+1)}{2!}e^{-x-2h} + \cdots\right] \quad (42)$$
$$= e^{-x}$$

and this definition as previously pointed out when $n = -1$ should be distinguished from the ordinary formula $de^{-x}/dx = -e^{-x}$ which has a negative sign.

Liouville then proceeded to use Eq. 41 together with the gamma integral to obtain the n^{th} generalized integral of x^n and obtained results which do not agree with those in Eq. 30. His results will be discussed later in this paper where the reason for the error is pointed out.

5. GENERALIZED DIFFERENTIATION

It was pointed out in connection with Eq. 30 that its use would be restricted to values of $n > 0$. This restriction was made since when n is negative in dealing even with such a simple case as the Null-Unit function $H(x, x_0)$ infinite values are encountered as $x \to x_0$ in the formula which may be written symbolically

$$\int_{x_0}^x H(x, x_0)dx^{-n} = \frac{(x-x)^{-n}H(x, x_0)}{\Pi(-n)}. \quad (43)$$

These infinite values can be avoided by using the method of Schwartz in which any function $f(x)$ is represented by its functional $f(\phi)$ as follows

$$f(\phi) = \int_{-\infty}^\infty f(x)\phi(x)dx. \quad (44)$$

Here $\phi(x)$ represents a set of continuous infinitely differentiable functions that are zero outside of a finite interval. It also follows

$$f^{(n)}(\phi) = f(\phi^{(n)}) = (-1)^{n-1}\int_{-\infty}^\infty f(x)\phi^{(n)}(x)dx. \quad (45)$$

It should be noted that in the Schwartz (3) method integration and differentiation are commutative. This is also the case in the method where the Null-Unit function $H(x, x_0)$ is used as in this paper and this property holds when the two methods are used together as will be done presently.

There are several different cases now to be considered and these will be considered separately for simplicity and in all cases the Null-Unit function will be used.

(a) Problems involving integral derivatives of the Null-Unit function

The functional representation of the Null-Unit function $H(x, x_0)$ is

$$H(\phi) = \int_{-\infty}^\infty H(x, x_0)\phi(x)dx = \int_{x_0}^\infty \phi(x)dx \quad (46)$$

and the functional representation of its n^{th} differential coefficient (n integer) as shown by Schwartz is

$$H^{(n)}(\phi) = H(\phi^{(n)}) = (-1)^{n-1}\int_{x_0}^{\infty}\phi^{(n)}(x)dx$$
$$= (-1)^{n-1}\,\phi^{(n-1)}(x_0). \qquad (47)$$

Here the functional representation of the n^{th} derivative exists although $H^{(n)}(x, x_0)$ which it represents does not exist in the mathematical sense.

(*b*) *Problems involving fractional derivatives of the Null-Unit function*

Let the $(n - m)^{th}$ derivative be required where n is an integer and m is a positive fraction $(0 < m < 1)$. This can always be readily arranged by proper choice of n. For instance if $(n - m) = 3.3$ then $n = 4$ and $m = .7$. Separate the problem into two parts first finding the m^{th} by the method in this paper and then the n^{th} differential coefficient by the method of Schwartz. First we have

$$\int_{x_0}^{x} H(x, x_0)dx^m = \frac{(x - x_0)^m H(x, x_0)}{\Pi(m)} = f(x). \qquad (48)$$

Then the Schwartz representation of the n^{th} differential coefficient of this quantity is

$$f(\phi^{(n)}) = \int_{-\infty}^{\infty}\frac{(x - x_0)^m H(x, x_0)\phi^{(n)}(x)dx}{\Pi(m)}. \qquad (49)$$

In evaluating this integral care must be taken when $n - m < -1$. Using as an example one given by Schwartz, take $n - m = -\frac{3}{2}$ so that $n = -2$

$$f(\phi^{(2)}) = \int_{x_0}^{\infty}\frac{(x - x_0)^{\frac{1}{2}}\phi^{(2)}(x)dx}{\Pi(\frac{1}{2})}$$
$$= \int_{x_0}^{\infty}\frac{(x - x_0)^{-\frac{1}{2}}\phi'(x)}{\Pi(-\frac{1}{2})} = \operatorname*{Lim}_{\epsilon\to 0}\int_{x_0+\epsilon}^{\infty}\frac{(x - x_0)^{-\frac{1}{2}}\phi'(x)dx}{\Pi(-\frac{1}{2})}$$
$$= \operatorname*{Lim}_{\epsilon\to 0}\left[\int_{x_0+\epsilon}^{\infty}\frac{(x - x_0)^{-\frac{1}{2}}\phi(x)dx}{\Pi(-\frac{3}{2})} + \frac{\phi(x_0)}{\Pi(-\frac{1}{2})\sqrt{\epsilon}}\right]. \qquad (50)$$

Here the first term is the functional representation of $(x - x_0)^{-\frac{1}{2}}H(x, x_0)/\pi(-\frac{3}{2})$ and is what M. Hadamard calls the finite part of the divergent integral and may be denoted by the symbol

$$f^{(2)}(\phi) = f(\phi^{(2)}) = \Gamma\int_{x_0+\epsilon}^{\infty}\frac{(x - x_0)^{-\frac{1}{2}}\phi(x)dx}{\Pi(-\frac{3}{2})}. \qquad (51)$$

The other term $\phi(x_0)/\Pi(-\frac{1}{2})\sqrt{\epsilon}$ corresponds to $1/\Pi(-\frac{1}{2})\sqrt{\epsilon}$ times a unit mass placed at x_0 in mass space. This part of the solution does

not exist as a function although its functional representation in mass space does exist. It is interesting to note the same type of additional term is obtained using the Heaviside method formally

$$\frac{d}{dx}\left[\frac{(x-x_0)^{-\frac{1}{2}}}{\Pi(-\frac{1}{2})}H(x,x_0)\right] = \frac{(x-x_0)^{-\frac{1}{2}}}{\Pi(-\frac{1}{2})}H(x,x_0) + \operatorname*{Lim}_{x\to x_0}\frac{(x-x_0)^{-\frac{1}{2}}}{\Pi(-\frac{1}{2})}\frac{dH(x,x_0)}{dx}. \tag{52}$$

Here of course $dH(x, x_0)/dx$ does not exist in the mathematical sense but its representation $\phi(x_0)$ does exist as already derived in Eq. 50.

(*c*) *Problems involving* $(n-m)^{th}$ *fractional derivative of* $(x-x_0)^p H(x, x_0)/\Pi(p)$

Let the $(n-m)^{th}$ derivative be required where n is an integer and m as in the previous example (*b*) is a positive fraction $(0 < m < 1)$.

Perform q integrations or differentiations on the function as may be required to obtain $(x-x_0)^{p+q}H(x, x_0)/\Pi(p+q)$ where $0 < p+q < 1$. If p is negative the Schwartz method in (*b*) should be used so as to be sure that terms corresponding to $\phi(x_0)$ in Eq. 50 are properly included. The m^{th} integral of this quantity is found using Eq. 33 and obtaining $(x-x_0)^{p+q+m}H(x, x_0)/\Pi(p+q+m)$. Then perform the operation of differentiating n times together with the inverse of the q integrations or differentiations first performed and obtain as a result $(x-x_0)^{p-n+m} \times H(x, x_0)/\Pi(p-n+m)$. If $p-n+m$ is negative this last operation should be done by the Schwartz method as in (*b*) obtaining

$$f(\phi^{(n+q)}) = \int_{-\infty}^{\infty}\frac{(x-x_0)^{p+q+m}H(x,x_0)\phi^{(n+q)}(x)dx}{\Pi(p+q+m)} \tag{53}$$

and this evaluated as in (*b*).

This changing from function space to mass space may appear quite complicated but it is only done where necessary to avoid difficulties at points where the function may tend to infinity and also to avoid the introduction of the impulse function and its derivatives. It is possible to do the work formally in the Heaviside manner if the method used in deriving Eq. 52 is followed although here of course terms such as $H^{(n)}(x, x_0)$ will be encountered which do not exist in the mathematical sense. Their representation in mass space $(-1)^n\phi^{n-1}(x_0)$ does exist.

6. LIOUVILLE'S RESULT FOR FRACTIONAL DIFFERENTIATION

In the introduction it was mentioned that it can be shown that the functions used in the definitions of Abel, Riemann, Heaviside, Bromwich and Carson are actually of the type $f(x)H(x, x_0)$. These formulas can be readily derived from those given above as will be shown in a later paper. It was also mentioned that a method due to Liouville gave a

different result. Liouville's result is:

$$\frac{d^p}{dx^p}\left[\frac{\Gamma(m)}{x^m}\right] = \frac{d^p}{dx^p}\int_0^\infty e^{-xk}k^{m-1}dk = (-1)^n\frac{\Gamma(m+p)}{x^{m+p}}. \qquad (54)$$

It will be noted that this result differs from that given in Eq. 30. Davis (5) refers to various discussions between mathematicians as far back as DeMorgan 100 years ago trying to decide in favor of one or both of these formulas. He concludes: "It is now apparent that there exist essentially two definitions of fractional operators which have different domains of usefulness. A great deal of confusion has arisen from this fact as may be seen from arguments advanced by Kelland in favor of formula [54] instead of [30]" and again he says, "This leads to the general observation that the Liouville formula $\left[{}_{-\infty}D_x^{-p}f(x)\left(=\int_{-\infty}^x f(x)dx^p \text{ in our notation}\right)\right]$ is applicable in the case where

$$f(-x) = 0(x^{\epsilon-p}) \qquad (\epsilon > 0) \quad [55]$$

and the Abel-Riemann formula ${}_0D_x^{-p}f(x)\left(=\int_0^x f(x)dx^p\right)$ when

$$f(1/x) = 0(x^{1-\epsilon}) \qquad (\epsilon > 0) \quad [56]$$

where the statement $f(x) = 0(\phi)$ has the customary meaning that $\lim_{x\to\infty} |f(x)|/\phi(x) =$ a constant."

This explanation, however, seems to overlook the fact that when x is negative the Laplace integral becomes infinite. Its value of $\Gamma(m)/x^m$ assumed above only holds when $x > 0$. This condition can be introduced into this relation by writing

$$\frac{\Gamma(m)}{x^m}H(x,0) = \int_0^\infty e^{-kz}k^{m-1}H(x,0)dk \qquad (57)$$

so that both sides are zero when $x < 0$. The situation on the right-hand side of this equation is now more complicated than that taken by Liouville since it is the product of two functions of x viz e^{-kz} and $H(x, 0)$. For the generalized differentiation of a product we must use a formula of the type given by Leibnitz (6).

$$\frac{d^p}{dx^p}(uv) = \sum_{r=0}^{r=\infty}\binom{p}{r}\frac{d^r u}{dx^r}\frac{d^{p-r}r}{dx^{p-r}}, \qquad (58)$$

where r takes integer values from 0 to ∞. In Eq. 58 let $u = e^{-kz}$ and

$v = H(x, 0)$, then

$$\frac{d^p}{dx^p}\left[\frac{\Gamma(m)}{x^m}H(x,0)\right] = \int_0^\infty e^{-kx}H(x,0)dk\left[\frac{x^{-p}}{\Pi(-p)}k^{m-1} - \binom{p}{1}\frac{x^{-p+1}}{\Pi(1-p)}k^m + \binom{p}{2}\frac{x^{-p+2}}{\Pi(2-p)}k^{m+1}\cdots\right]. \quad (59)$$

Or (7)

$$\frac{d^p}{dx^p}\left[\frac{H(x,0)}{x^m\Pi(-m)}\right] = \frac{H(x,0)x^{-m-p}}{\Pi(-m)\Pi(-p)}\left[1 + \binom{p}{1}\frac{(-m)}{1-p} + \binom{p}{2}\frac{(-m)(-m-1)}{(1-p)(2-p)} + \cdots\right]$$

$$= \frac{H(x,0)x^{-m-p}}{\Pi(-m)\Pi(-p)}F(-p, m, -p+1, 1) \quad (60)$$

$$= \frac{x^{-m-p}H(x,0)}{\Pi(-m)\Pi(-p)}\frac{\Pi(-p)\Pi(-m)}{\Pi(-m-p)} = \frac{x^{-m-p}H(x,0)}{\Pi(-m-p)}$$

This agrees with the result previously obtained and does not give a second definition for the fractional operator as stated by Davis.

It may be well to point out that in using the Leibnitz formula in this derivation we interpreted $d^{p-r}H(x,0)/dx^{p-r}$ as $\int_0^x H(x,0)dx^{-p+r}$ since p is not assumed to be an integer, and d^re^{-x}/dx^r was taken as the ordinary derivative since r is an integer.

It should be apparent that if we try to derive Liouville's result we should interchange our assignment of u and v, making now $u = H(x, 0)$ and $v = e^{-kx}$. From analogy with Eq. 40 he then interpreted $d^{p-r}e^{-x}/dx^{p-r} = (-1)^{p-r}e^{-x}$ and we have pointed out in Eq. 41 this is not justifiable if the limits are taken to be $-\infty$ and x. In addition, he neglected the presence of $H(x, 0)$ which when differentiated an integral number of times (r an integer) gives rise to a series of impulse functions so that if he recognized the presence of $H(x, 0)$ he would at least have written

$$\frac{d^p}{dx^p}\left[\frac{\Gamma(m)H(x,0)}{x^m}\right] = (-1)^k\frac{\Pi(m+p)}{x^{m+p}} + \sum_{r=1}^{r=\infty} f_r(x)\frac{d^rH(x,0)}{dx^r}. \quad (61)$$

Such an expansion in terms of impulse functions is of course illegitimate in the mathematical sense since these impulse functions do not exist. This question, however, is an interesting one and will be discussed further in a later paper. It may be well to point out that when p is an integer Leibnitz's formula consists of only a finite number of terms and the correct result is obtained as has been noted by others.

CONCLUSIONS

A problem which continually recurs in practical problems of engineering and physics, is how to perform differential operations when sudden discontinuities are involved. The usual mathematical methods bring in derivatives of the impulse function, which "do not exist" in a

mathematical sense. Many solutions of such problems, obtained by various expedients that disregard the impulse function derivatives, have been used. But uncertainties have remained, as witness Liouville's relation shown above to be incorrect despite its general acceptance for over 100 years. The root of the difficulty lies in the usual practice of simply stating an equation is valid only when $x > x_0$, and disregarding the effects of the discontinuity at $x = x_0$.

All of these difficulties are avoided if the Null-Unit function $H(x, x_0)$ is employed as a multiplier to indicate these discontinuous properties, and its derivatives are interpreted by the method of Schwartz. This use of $H(x, x_0)$ makes generalized integration and differentiation completely reversible and eliminates the need of determining constants of integration.

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(2) J. J. SMITH AND P. L. ALGER, "A Derivation of Heaviside's Operational Calculus Based on the Generalized Functions of Schwartz," *Trans. AIEE*, Vol. 68, Part 2, 1949, pp. 939–944.

(3) L. SCHWARTZ, "Generalization de la Notion de Function, de Derivation, Etc.," *Annales*, l'Universite de Grenoble, Vol. 21, p. 57, (1945). Also, *Annales des Telecommunications* (Grenoble, France), Vol. 3, Number 4, April, 1948, pp. 1–6.

(4) JOSEPH LIOUVILLE, "Memoires sur quelques questions de Geometrie et Mechanique et sur le Calcul des differentielles a indices quelconques," *Journal de L'Ecole Polytechnique*, 1832, pp. 70–182.

(5) See reference 1, p. 69.

(6) Formula of Leibnitz. Reference 1, p. 154.

(7) See "Modern Analysis," by Whittaker and Watson. (Cambridge University Press, 1927), for the value of the Hypergeometric Function $F(a, b, c; 1)$, p. 282.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

HUMAN RESPONSE STUDIES *

In the belief that the determination of the optimum characteristics of aircraft controls could be effectively studied by measuring human response to typical visual stimuli, the Air Research and Development Command requested that the Laboratories make a study of human frequency response.

Conventional methods for obtaining frequency-response characteristics do not appear to be applicable in the study of human visual-motor activity. The human does not represent a physical system that can be described by a set of differential equations of known coefficients, which would allow the determination of the frequency response. Also, one cannot compute human frequency-response characteristics by observing a pilot's response to a stimulus that follows a sinusoidal pattern since the predictability of the sinusoid would enable him to respond according to pre-knowledge—at least without depending wholly upon the visual stimulation—and the realism of the typical tracking problem would be lost. Nor can the frequency response for the human reasonably be obtained by using a transient visual input, for an analysis of this sort would then assume that his response to (say) a step function could be reconstructed by summing his responses to the frequencies in the Fourier analysis of that function. It is therefore necessary to study the human operator by a technique that is different from these three methods.

Of course, any technique which is used to study human motor responses must be selected to yield pertinent information on those characteristics; that is, information that is not only valid but is, as well, couched in terms or expressions which are native to the frame and scope of the primary problem. As an example, if a human is to be part of a complex electromechanical system, it would be indispensable to a complete knowledge of the system to know how the phase and amplitude of his output vary as functions of frequency—in other words, some form of a frequency-response characteristic would be pertinent. Pursuing the example, which is actually the basis for this problem of the response to visual stimuli by humans in control of aircraft, the frequency dependence of phase and amplitude of a pilot's output can be determined from measurements of the cross-spectral density of the output and input, and the spectral density of the input signal. The frequency dependence of amplitude alone can be obtained by measuring the input spectral

* Human Frequency Response Studies are conducted at The Franklin Institute Laboratories under Contract AF 33(038)-10420 with the United States Air Force.

density and the output spectral density. Then, if the pilot's response is approximately linear with constant coefficients, over a certain bandwidth, the ratio of the output spectral density to the input spectral density is the amplitude part of the human operator's frequency response. The complex frequency response is the ratio of the cross-spectral density of output and input to the spectral density of the input.

Charged with performing these studies, the Electrical Engineering Group built a so-called simplified airplane simulator, consisting of a bucket seat, joy stick, and an oscilloscope mounted at eye-level before

FIG. 1. Simplified airplane simulator.

the pilot. This device was used in preliminary experiments to indicate whether the method of spectral density analysis is applicable to the study of human tracking responses in the larger and more complicated dynamic airplane simulator, also constructed in these laboratories. The visual stimulus, or input, is presented as a series of step deflections in the horizontal dimension on the oscilloscope screen. The program of deflections of the pip consists of a time series of pulses whose average number of crossings is determined by sampling a Poisson distribution. Frequencies of from zero to about three cycles per second constitute this statistical input. The pilot's task is to manipulate the controls to keep

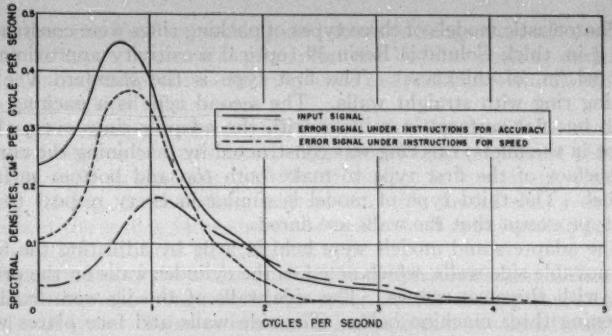

FIG. 2. Spectral density comparisons.

the pip on a central vertical reference line; the error signal is the instantaneous position of the pip, and diminishes as the pilot restores the pip to the vertical reference line.

A series of experiments was conducted to observe how the spectral densities of error, input, and output functions vary as functions of instructions to, and training of, the pilot, and of the amplitude of the input signal. In one experiment subjects were instructed to track either with greatest possible accuracy, or with greatest possible speed. Changes in the spectral densities of the error signals for one well trained subject acting according to these two commands are given in the graph. The program was the same for both the speed and accuracy trials.

EZRA S. KRENDEL

SEALING ACTION OF PACKING RINGS

The Mechanical Engineering Group, at the request of Raybestos-Manhattan, Incorporated, undertook the task of determining the manner in which pressure is transmitted through "VeeFlex" packing rings, an R-M product. In addition it was to be decided by appropriate tests which of two types—straight-wall or flared-wall VeeFlex rings—had the better sealing properties, and whether additional tests would be necessary to fully describe the sealing action of these rings.

Photoelastic analyses were made to observe the way in which pressure is transmitted through the rings to the cylinder walls, and radial pressure tests were performed to determine the magnitude and distribution of pressure where the face of the packing ring bears on the cylinder wall when the ring assembly is compressed by a packing nut. These latter tests were made of two sets each of three rings (of both straight- and flared-wall type), at three locations on the circumference of the middle ring of each set.

Photoelastic models of three types of packing rings were constructed from ¼-in. thick Columbia Resin-39 (optical sensitivity approximately 86.6 psi./in. of thickness). The first type is the standard VeeFlex packing ring with straight walls. The second type is a packing ring which has flat surfaces in contact with the adapter rings everywhere except in the fillets; this ring was constructed by machining the curved top surface of the first type to make both top and bottom surfaces parallel. The third type of model is similar in every respect to the first type except that the walls are flared.

The adapters and models were held in a jig by adjusting the fixed and movable side walls, which acted as the cylinder walls on the prototype, with three set screws. The side walls of the jig were fixed by tightening three machine bolts. The side walls and face plates were made from lucite. A window, through which the photoelastic model was observed, was machined in the face plates, so that the overlapping lucite would not influence the stress patterns. The assembly was then

FIG. 1. Straight-wall VeeFlex packing. × 14.

FIG. 2. Straight-wall packing with flat surface. × 14.

put into a loading frame subjected to a compression load, and the resulting stress patterns photographed with monochromatic (mercury-vapor lamp) light.

The first model with straight walls was assembled in the jig with passive support of the walls. Figure 1 shows the resulting stress pattern for axial loading. The photograph shows that the stress pattern exists at the sealing face of the packing, due to the bending effect of the packing against the simulated cylinder walls.

The second model was assembled in the jig with passive support of the walls and loaded axially. Figure 2 shows the stress patterns in this model. It can be plainly seen that very little stress exists near the sealing surfaces, which would tend to indicate that this type of packing has inferior sealing properties. This conclusion is not entirely correct when it is realized that many simple rectangular-cross-section packings are in use and are rendering good service. The reason for

this is that the VeeFlex packing, and those packings with full contact, seal by different mechanical principles: the VeeFlex packing seals due to the bending deflection of the ring while the packings with full contact seal due to cubical dilatation.

The third and last photoelastic model, with flared walls, is intended to show influence of flared faces. The model was compressed between the side walls so that the total flared surface is in contact with the walls. In Fig. 3, the stress pattern for this model shows definite stresses at the sealing faces and tends to denote that the sealing properties exceed those of the straight-wall type.

Photographs of the stress patterns in the photoelastic models of the packing rings show highest fringe order at the sealing faces in the flared-wall type. This would seem to indicate that the flared-wall type seals more fully than the straight-wall type. However, since the difference in modulus and Poisson's ratio between the CR-39 and the

FIG. 3. Flared-wall packing. × 29.

packing material has not been taken into account, definite conclusions cannot be stated. The photographs of the model with flat surfaces in contact indicate that there is very little sealing action at the faces in contact with the walls.

All radial pressure tests were performed on rings with 3-in. outside diameter and 2-in. inside diameter. The packing rings were assembled in stacks of three on a jig that allowed no lateral deflection on the inside diameter. Another jig was placed around the packing ring assembly eliminating lateral deflection on the outside diameter. The theory of the tests is that leakage will occur when the applied radial pressure exceeds the lateral pressure of the packing due to compression of the assembly by the packing nut. The pressure to cause leakage was measured at several locations along the face between lip and heel on the middle packing ring in a stack of three rings. The top edge of the groove in the outside jig was located and measured relative to the lip of the packing with a cathetometer. A compression load was applied to

the packing assembly through three ½-in. square steel bars located approximately 120 deg. apart. Preliminary tests showed that three loads of 100-lb. increments were sufficient to reveal the effect of varying compression loads on the assembly. The vertical deflections of the stack of packing rings under these loads were measured by means of two Federal dial gages located approximately 180 deg. apart. The loads were applied and after a sufficient lapse of time to allow for complete deflection of packing assembly, oil pressure was applied by means of a manually operated pump to the packing rings via the groove in the outside jig. Because of the extreme thinness of the oil film that escaped between the jig and packings, the exact pressure and location of leakage was difficult to ascertain. This problem was overcome by releasing the pressure completely each time and allowing air bubbles to seep back into groove before next increment of load was added. The leakage of the air bubbles allowed more positive observation. The pressures to cause leakage were recorded for each load at every location of the groove on the packing ring. Two sets each of the straight-wall and flared-wall types were tested.

A general common trend is evident for the pressure distribution across the faces of both the straight-wall and flared-wall packings. Both types seem to give maximum sealing performance at a point on the face slightly below mid-width, and little difference is noted as to the sealing properties at the lips of the two types on the basis of the tests performed. Variations are evident in the peak pressure values at different locations on the packings. These variations were caused, not by leakage past the ring as a unit, but mainly by seepage through irregularities in the material of the packings. The magnitudes of peak pressures do not seem to be in direct proportion to the compression loads as would be expected. In many cases a compression load of 100 lb. on the assembly did not provide a complete seal between the packings and the cylinder walls. The pressure required to break the seal was more than doubled when the compression load on the assembly was increased from 100 to 200 lb. At 300-lb. compression, the pressure necessary to rupture the seal was not one and one-half as great as that necessary for the 200-lb. load.

On the basis of these tests, it can be concluded that flared faces have negligible effect on magnitude and distribution of pressure across the walls of the packing rings. This refutes the photoelastic results but, as stated before, the photoelastic method on this type of structure should be examined with caution.

It can also be concluded that for the majority of cases, leakage does not occur past the packings as a unit, but occurs through local irregularities in the material of the packings. This explains the large variations of sealing pressures in the radial pressure test curves.

A. O. BERGHOLM AND P. W. SWARTZ

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

A MAGNETIC TAPE "MEMORY" FOR SEAC

A computer-controlled external auxiliary "memory," using magnetic tape as the recording medium, is extending the problem-solving capacity of SEAC, the National Bureau of Standards Eastern Automatic Computer. Recently developed by James L. Pike of the NBS computer laboratory, the new magnetic tape memory unit combines high-speed starting, stopping, and reversing with notable mechanical simplicity. Two of the units are now in routine operation with SEAC, and plans call for adding two more.

In the new system, the magnetic tape rests lightly on two smooth-surfaced rollers that rotate continuously but in opposite directions. Between these two rollers the tape passes through magnetic heads for recording, pickup, and erasing. When either of two control solenoids is energized, a low-inertia rubber-covered roller presses the tape against one of the smooth rollers. This quickly starts the tape moving in the desired direction. Tape inertia is kept low by letting each end fall in loose folds into a tank or bin. Each tank consists of two plates of glass spaced just a little more than the width of the tape.

Dedicated in June 1950, SEAC was the first postwar automatically-sequenced super-speed computer to go into productive operation. The all-electronic machine was developed and constructed by the NBS staff under the sponsorship of the Department of the Air Force to provide a high-speed computing service for application of mathematical techniques to large-scale problems of military procurement and administration. For more than a year now, SEAC has been actively turning out solutions to these and other important problems.

An electronic digital computer such as SEAC has a limited storage capacity in its internal high-speed memory, in which partial solutions are stored until needed later in the course of the solution of a problem. To handle many types of problems, it therefore becomes necessary to store information in an external memory; information is transmitted out of the computer for storage, and fed back in later in the problem. The speed with which this can be accomplished is a very important factor in the over-all operating speed. For instance, when Teletype tape was used with SEAC for auxiliary storage as a temporary expedient, reading the information onto the tape and back again often took more than 95 per cent of the total problem-solution time.

Thus far, the most promising method of reducing this time has been the use of magnetic recording. However, while computer-controlled

* Communicated by the Director.

auxiliary memory systems using magnetic pulses have been employed successfully in computers other than SEAC, most of these systems have required complex and expensive mechanisms to start, stop, and reverse the magnetic tape or wire with the necessary speed. The NBS magnetic tape memory has the advantage of combining speed with simplicity and economy.

The speed and simplicity of the NBS system result from the successful elimination of reels and servomechanisms. Only two small masses need to be accelerated in starting the tape, the small jam roller and a few feet of tape hanging into the tank.

Although the tanks are big enough to hold several thousand feet of tape, an ordinary 1200-foot spool is ample for most problems. The tanks are enclosed on all edges and have slots in the top for the tape to enter. Because the tanks are just wide enough to clear the tape, the loose folds in which the tape falls have no tendency to turn over or to become tangled.

Several problems have been encountered in developing the tape memory mechanism. For one thing, the tape tends to acquire an electrostatic charge as it passes through the drive mechanism. This can become quite troublesome at higher speeds, causing the tape to cling to the walls of the tank as soon as it leaves the drive mechanism. If the charge is strong enough, the tape may continue to stick at the top of the tank until it backs up into the mechanism and is damaged by a sharp fold. The present solution—satisfactory at moderate speeds (up to 8 ft. per sec.)—is to ionize the air where the tape leaves the drive unit, using strips of alpha-emitting polonium. Another possible answer to the problem, though somewhat inconvenient, would be to control the humidity within the tanks. The ideal solution would probably be to make the tape base material sufficiently conductive, so that a charge could not collect. Experiment indicates that base material having a resistance of not more than a few megohms per unit square should be satisfactory.

A second limitation on the operating speed of the tape is imposed by the need to erase information from the tape. The necessary erase frequency increases as the tape speed increases. A tape speed of 16 ft. per sec., for instance, requires an erase frequency of several hundred kilocycles. Because of hysteresis losses, ordinary tape heads cannot operate at such high frequencies. New heads with powdered iron magnetic circuits, now being developed for use at much higher frequencies, should solve this problem. Meanwhile, it is always possible to erase the tape in a separate operation, at less than the normal running speed.

A third major problem in using magnetic recording in computer work is the presence of flaws in the magnetic tape. Commercially available tape has many small imperfections in the magnetic oxide

coating, which are quite undetectable in ordinary audio work. In the recording of computer pulses, however, the loss of magnetic signals over a very small area in the tape may mean the loss of one or more digits of information. And in computer work the loss of a single digit—which may occupy less than 0.020 in. on the tape—cannot be tolerated. Some specially-treated tapes now available are nearly free from flaws; and improved manufacturing techniques may soon eliminate such difficulties. Meanwhile a process of removing the imperfections by scraping the recording surface over a suitably shaped knife-edge has been developed. This is somewhat laborious, but is used as a temporary expedient. Another method of avoiding the flaw problem is to record from the back side of the tape through the plastic base. Although this allows only about half as much information to be recorded in a given length of tape, the result is unaffected by flaws in the coating.

NOTE: For other information on SEAC, see "SEAC, the National Bureau of Standards Eastern Automatic Computer," NBS *Technical News Bulletin*, Vol. 34, p. 121 (Sept. 1950); also, "SEAC Demonstrates High Reliability," NBS *Technical News Bulletin*, Vol. 35, p. 72 (May 1951).

IGNITION OF FIBROUS MATERIALS BY SELF-HEATING

Recent investigations by the fire protection laboratory of the National Bureau of Standards have provided conclusive proof that closely packed fibrous materials can ignite by self-heating. NBS tests show that these materials in quantity, after standing for some time, may develop an internal temperature higher than the surrounding or ambient temperature. The extent of this temperature rise depends upon the material, its density and packing, and upon the ambient temperature and the length of time this state is maintained. This work thus gives strong evidence that suitable precautionary procedures should be taken in the storage and use of such commodities. For instance, a 12-in. cube of wood fiberboard kept at 147 F. develops a peak interior temperature of 148 F.; a sample kept at 210 F. develops a maximum internal temperature of 230 F.; and a sample at 240 F. develops a temperature of 305 F. If the size of the specimen is increased, the amount of self-heating is increased.

In general, the effect of self-heating is small; in some special cases, however, serious consequences may result if this effect is overlooked, particularly where materials packed in large stacks are involved. In one case, nine carloads of insulation fiberboard were shipped from a factory in the South, bound for New York State, before the heat of fabrication was completely dissipated. Seven days later one carload was discovered to be afire. The other eight cars when unloaded into an Army warehouse after 10 to 12 days on the road, were stacked in a single pile of more than 24,000 cu. ft. Four days later the warehouse and its contents were destroyed by fire, the loss amounting to 2½ million dollars.

During the investigation of the cause of the fires, the possibility of ignition due to self-heating was considered. As a result, The Corps of Engineers asked the National Bureau of Standards to investigate whether self-heating in the interior of wood fiberboard could be carried to such an extent as to cause the material to ignite.

Mr. A. C. Hutton and Mr. D. Gross of the NBS Fire Protection Section developed equipment and methods for studying self-heating characteristics over a wide range of size and temperature conditions. Boards cut octagonally were stacked in depths equal to the width of the octagon. These were introduced into a suitable oven or furnace and held at constant ambient temperature until the peak of self-heating was passed or until the self-heating had resulted in ignition. Series of increasing ambient temperatures were used until the lowest ambient temperature at which ignition would occur was found.

The initial tests were made on wood fiberboard specimens 0.01 to 12 in. in diameter. The larger the volume of the specimen, the greater is the self-heating temperature rise within the specimen for a given ambient temperature and the lower is the external temperature required to initiate combustion. Ignition was found to start at the geometrical center of the specimen and spread outward through the remainder of the material.

Data were also obtained on cane fiberboards, felted cotton linters (alpha cellulose), Douglas fir, crushed coal, and alfalfa hay. For the cellulose-type materials so far tested, the lowest ambient temperatures that cause ignition fall within a range of 50 degrees F. for a given sample size, but the times required to reach ignition vary widely. Thus, a 2-in. sample of wood fiberboard at 356 F. ignites in 18 min., while a 2-in. sample of felted cotton fiber ignites at 394 F. after 63 hr. When the ambient temperature is allowed to rise with the self-heating of the specimen rather than being kept constant, ignition is accelerated.

For a given material, the logarithm of the ignition temperature was found to vary inversely as the logarithm of the specimen size. It appears probable that extremely large volumes of fibrous materials will ignite at relatively low ambient temperatures and that a potential fire hazard exists in the transportation and storage of these materials—in particular, immediately after hot processing and drying. More effective control of shipping and storing temperature and of stack size is therefore recommended to reduce the probability of ignition due to self-heating.

A SIMPLIFIED RADIOACTIVITY SURVEY INSTRUMENT

Portable self-contained radioactivity survey instruments of the gamma type generally use a microammeter to indicate the radiation level. Ideally these survey instruments should be low in cost as well

as compact and rugged. Yet microammeters are inherently neither cheap nor rugged, nor are they suited to mass production in extremely large volume. A new gamma survey instrument, recently developed by S. R. Gilford and S. Saito of the National Bureau of Standards electronic instrumentation laboratory, requires no microammeter.

In the NBS instrument, which was developed for the Navy Bureau of Ships, radiation levels are read directly from a potentiometer dial. To read an unknown value of radiation, the operator turns the dial to the point at which an audio oscillation just begins. This point is determined aurally with the small earphone that is standard equipment for most survey instruments. This aural indication is particularly convenient for plotting contours of equal radioactivity; with the dial set for a particular radiation level, it is an easy matter for the operator to walk along and locate aurally a series of equally radioactive points.

Like many other gamma-radiation survey instruments, the NBS device uses a detector tube of the halogen-filled GM type, together with a vibrator high-voltage power supply operating from flashlight-type batteries. Output current of the detector is proportional to the incident radiation.

The heart of the aural indication method is a thyratron relaxation oscillator circuit. If the potential difference between the grid and cathode of the thyratron exceeds the firing potential, the circuit will oscillate. The unknown voltage derived from the detector tube current is applied to the outer terminals of the potentiometer, while the thyratron grid is connected to the moving contact. The potentiometer setting at the threshold of oscillation thus depends on the radiation level, and the potentiometer dial can be calibrated directly in radiation units.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

February 20, 1952

The Stated Monthly Meeting of The Franklin Institute was held on February 20, 1952, in the Lecture Hall of The Franklin Institute. Mr. S. Wyman Rolph, President, called the meeting to order at 8:25 P.M. Approximately 225 persons attended.

The President stated that the Minutes of the Stated Monthly Meeting for December were printed in full in the January issue of the JOURNAL; there being no corrections or additions they were approved as printed. He stated further that the Minutes of the Annual Meeting in January were being printed in full in the February issue of the JOURNAL, which had not yet been mailed to members; approval was, therefore, postponed until the March meeting.

The Secretary reported the following elections to membership for the month of January:

Active	33
Associate	22
Student	15

and reported further that total Institute membership as of January 31, 1952, was 6,256.

He announced the award of a Citation to The Franklin Institute on February 19th by the Philadelphia Chapter of the National Society of Professional Engineers, representing the Engineering Societies in Philadelphia. President Rolph, who had accepted the award on behalf of the Institute, read the citation: " . . . for notable promotion of advances in Engineering, to wit: For establishing the first school for instruction in Mechanical Arts—For publishing since 1825, THE JOURNAL OF THE FRANKLIN INSTITUTE—For the unique construction and public display of Mechanic Arts Exhibits—For its extraordinarily fine Technical Library—For its modern laboratories, and the significant contribution to Military and Industrial Research and Development—For its continuing recognition of outstanding contributions to engineering progress, through the activities of its Committee on Science and the Arts."

The President then introduced the speaker for the evening—Dr. W. F. G. Swann, Director of the Institute's Bartol Research Foundation, and Senior Adviser to The Franklin Institute Laboratories for Research and Development. Dr. Swann's lecture on "The Nature of Research in Physics" was favorably received by the audience, and his frequent humorous undertones were also fully appreciated.

Because of the lateness of the hour, the usual question-and-answer period was eliminated, and the meeting adjourned at 10:00 P.M. with a rising vote of thanks to Dr. Swann.

HENRY B. ALLEN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, February 13, 1952.)

HALL OF THE COMMITTEE,
PHILADELPHIA, FEBRUARY 13, 1952.

MR. HOWARD STOERTZ in the Chair.

The following report was presented for final action:

No. 3246: Franklin Medal.

This report recommended the award of the Franklin Medal to Wolfgang Pauli, of Zürich, Switzerland, "In consideration of his work toward the understanding of atomic physics and specifically for his formulation of the exclusion principle."

JOHN FRAZER,
Secretary to Committee.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS

February 20, 1952

ACTIVE FAMILY

Albert R. Latven — Thomas M. O'Neill — Aaron B. Sloane

ACTIVE

Wolcott Beard	Winfield Givens	Catherine A. Murray
Francis J. Bouda	W. Earl Graham	Thomas W. Pierie
Edward S. Brumberger	Grace Murray Hopper, Ph.D.	Joseph L. Pollard, Jr.
Cole Coolidge	Cardeen C. Lowry	Harrie B. Price, Jr.
F. S. Crysler	Sidney Margolis	Edwin D. Rattigan
Robert W. Drehmann	M. H. McCauley	Gerald N. Roberts
W. A. Fox	Albert McClenaghan	Rudolph M. Spitzer
George F. Francis		Morton L. Wallerstein, Jr.

ACTIVE NON-RESIDENT

Dennis Lee McElroy — H. Gale Heath, Joseph Pursglove, Jr. — Howard S. Turner, Ph.D.

NECROLOGY

Samuel S. Evans '50	Ernest Mather '43	H. Irvin Peter '50
Walter A. Lausterer '49	James C. Newlin '36	Edward Ray Strayer '45

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

SILBERSTEIN, LUDWIK: Generalities Concerning "Simple" and Mixed Photographic Emulsions as Manifested by Their Characteristic Curve.

ZADEH, LOFTI A.: A General Theory of Signal Transmission Systems.

BLADEL, JEAN VAN: Field Expandability in Normal Modes for a Multilayered Rectangular or Circular Wave-Guide.

GERSON, N. C.: The Colloquium on Auroral Physics.

MOON, PARRY AND DOMINA EBERLE SPENCER: Separability in a Class of Coordinate Systems.

BISPLINGHOFF, R. L. AND C. S. DOHERTY: Some Studies of the Impact of Vee Wedges on a Water Surface.

SAIBEL, EDWARD: Buckling of Continuous Beams on Elastic Supports.

RYDER, FREDERICK L.: Network Analysis by Least Power Theorems.

MUSEUM

One of the most spectacular growths of an industry has been in progress during the past decade. Ten years ago a $500 million a year industry was supported by the fragile vacuum tube which had few uses outside of radio. Electronics have found so many applications in tubes, television, computers, industrial controls, etc., that they are now estimated to sustain a $2.5 billion a year industry, with further expansion in sight. The principal cause of the increased production was the television set, which, on the average, uses twenty electronic tubes.

Our Museum is fortunate in possessing exhibits illustrating the two extremes of this growth, together with many intervening steps. Through the foresight and energy of a member, Mr. Leslie M. J. Woods, it was possible to obtain from Sir Ambrose Fleming a specimen of the first (two element) rectifying vacuum tube, and an original Lee De Forrest "Audion," the first three element tube which rendered possible the radio broadcasts we know today. Both these precious prototypes are authenicated by letters from their famous makers. It was upon knowledge derived from these tubes that the electronic industry has been built until recently.

The latest device, the transistor, performs most of the functions of the vacuum tube but it operates upon a different electronic principle. This new principle came to light when Bell Laboratory engineers were investigating the properties of semi conductors. Instead of having the compact atomic structure of good conductors of electricity, these have a looser structure in which some electrons have a tendency to wander from the outer orbits of the atoms. It was learned that, by placing two hair-thin wires in close contact to a small block of germanium a small positive charge on one wire, called the emitter, displaced electrons in the germanium between the two points of wire, causing other electrons to slip into the cavities thus created and producing a current between the wires which can be drawn off one, called the collector. Since more electrons enter the current than were in the original charge provided by the emitter, the result is an amplifying effect similar to that created in the vacuum tube.

The transistor has some advantages over the vacuum tube. It is small (peanut size), it has no vacuum, no glass tube, no fragile filament, no grid or plate. It has nothing to heat, nor anything to burn out. The transistor fits ideally into the mounting development of automatic electronic controls, computers, communication systems, and many other uses.

Visitors to the Museum should not fail to see the transistor exhibit in the Bell System room. Not only are its operating parts shown on a greatly magnified scale, but it becomes clear to the observer how and why it has advantages over the conventional vacuum tube. In issuing an invitation to members to visit this particular room we are confident in saying that it will be a rewarding experience, for the transistor exhibit is but one of the many fascinating exhibits provided to illustrate the part played by the telephone in our daily lives.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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that the action of armor plate under the impact of projectiles is much the same as subjecting the material to enormous hydrostatic pressures. "Studies in Large Plastic Flow and Fracture" is the compilation and condensation of all the significant papers that he has written in his many years of research; some have been reproduced completely while only the important material from others has been retained. A bibliography of all of Professor Bridgman's papers on the subject is given in the introduction.

The volume has been divided into four parts: Part I, "Tests under Hydrostatic Pressure"; Part II, "Other Tests Involving Large Deformations"; Part III, "Plastic Flow and Fracture after Prestraining"; and Part IV, "General Survey." In Part I, such tests as the tension of steel and other materials under pressure, two-dimensional tension of steel under pressure, compression and Brinnell hardness under hydrostatic pressure, and the collapse of thick hollow cylinders under external pressure are described in detail. In Part II, simple compression including volume changes in the plastic stages on various materials as soapstone, marble, copper, and steel; two-dimensional compression; torsion combined with simple compression; and shearing combined with hydrostatic pressure are discussed. In Part III, a variety of tests after prestraining, such as tension after tension prestraining, tension after compression prestraining, and compression after two-dimensional compression prestraining are considered. In Part IV, archaically titled "Gathering up the Threads," a summary of the significance of Professor Bridgman's research is given.

Generally, the book is quite detailed and could be improved by editing. It appears rather disorganized, is frequently wordy, and the transition from one subject to another is not always smooth. Undoubtedly, this is largely due to the fact that much of the book is simply a reproduction of many of the reports submitted to Watertown Arsenal and the NDRC. While perhaps interesting from an academic viewpoint, the lengthy descriptions of many of the experiments may not appeal to the average student. What is important is the significance of the results. And in reviewing the book from beginning to end, it was frequently difficult to find such significance. However, in the final part, Part IV, this adverse criticism is largely invalidated, for in a fine, philosophical dissertation, Professor Bridgman discusses the significance of his researches on plasticity and fracture of materials, and describes the fundamental characteristics of each. For one concerned with the basic behavior of materials under stress, or in fact, simply interested in the strength of materials, this section will be very worthwhile.

E. W. HAMMER, JR.

ADVANCED THEORY OF WAVEGUIDES, by L. Lewin. 192 pages, diagrams, 14 × 22 cm. London, Iliffe & Sons, Ltd., distributed in U. S. by British Book Centre, Inc., 1951. Price, $6.75.

The development of radar to meet the demands of the recent conflict was characterized by the steady exploitation of higher and higher frequencies. This in turn provided a stimulus for the application of a phenomenon previously of theoretical interest only, that is, the propagation of electromagnetic energy in hollow pipes as waveguides. Under security restrictions the war-time laboratories made great advances in both the theory of waveguides and their engineering development. Often the latter preceded the former, under the pressing necessity to get radar systems into service.

There has now been time to organize much of this war-time knowledge in orderly fashion, and to publish many articles and books on the theory and practice of waveguides and waveguide devices. Hence, one may question the necessity of still another book on the subject. In the present case, the answer lies in the fact that the book's title is perhaps misleading, and might better have been "Advanced Theory of Discontinuities in Waveguides." The work is restricted to obstacles, etc., and does contain material hitherto available only in scattered papers or unpublished laboratory reports.

After using Chapter I to provide a compact statement of electromagnetic theory, illustrated by its application to propagation in uniform rectangular waveguides, Mr. Lewin has devoted the balance of his book to an exposition of the theory of simple posts and diaphragms, tuned posts and tuned windows, steps, T-junctions, tapers, radiation from guides, loaded and corrugated guides. The treatment is highly mathematical, making considerable use of the

methods developed by Schwinger. Nevertheless, it is logically presented and carried through with clarity. The presentation is restricted to rectangular guides but serves to demonstrate methods which may be extended by the reader to other forms. The text is not meant as a design handbook, but rather as a demonstration of the theoretical analyses which may be used to develop formulas such as may be found in handbooks. For many engineers engaged in the design of waveguide systems, the present book will be unnecessary, but for those who are confronted with problems of developing new devices, or who wish to learn the source of the formulas they use in order to better understand their application, it will be valuable.

The author makes use of the Hertzian vectors in his fundamental theory. The advantage of this formulation over the more familiar formulation in terms of the vector potentials is not apparent here, since sinusoidal quantities with exponential notation are introduced at the outset, with the result that the vector potential and the Hertzian vector differ only by an imaginary factor. Indeed, the magnetic Hertzian vector as defined by Mr. Lewin is really a vector potential, since he omits the imaginary factor arising from the time differentiation. In a footnote he explains his choice in this case by stating that it gives different units for the electric and magnetic Hertzian vectors.

This brings us to another minor difference of opinion. Throughout the text the units are those of the cgs practical system, resulting in equations peppered with numerics. This might have been avoided if the mks rationalized system had been used, in which system the Hertzian vectors, symmetrically defined, have the different units of volt meters and ampere-turn meters. Anyone interested in making a comparison of the systems of units will find a similar development of electromagnetic theory from the Hertz vectors, using mks units, and applying it to waveguide problems, in Chapter 7 of "A Survey of the Principles and Practice of Waveguides," by L. G. H. Huxley.

Mr. Lewin has also provided an extensive bibliography of the periodical literature, conveniently classified in seven categories.

ALAN C. BYERS

FUNDAMENTAL MECHANISMS OF PHOTOGRAPHIC SENSITIVITY, edited by J. W. Mitchell. 347 pages, diagrams and illustrations, 20 × 26 cm. London, Butterworth Scientific Publications; New York, Academic Press, Inc.; 1951. Price, $9.50.

A symposium on photographic sensitivity was held at the University of Bristol in 1950. This book is a collection of the papers presented at that symposium. These papers almost all represent original experimental work, by a variety of workers in the U. S., England, and Europe. Most of the papers are complete, although several apparently did not make the deadline and are represented only by resumes. No discussion of the paper is included. By nature almost all of these papers are experimental, and as such deal with only restricted phases of their subjects. The one outstanding exception is a paper by J. W. Mitchell, in which he reviews the present status of latent image theory.

The papers are arranged roughly into five groups although there are many dealing with isolated topics that really do not fit into any one of the five groups. The first of these groups concerns itself with physical properties of the silver halides. Included are papers on F centers in alkali halides, diffusion processes in silver halides, optical and other physical properties. The second group deals with the production of silver halide grains and photographic emulsions. The kinetics and the inhibition of grain growth, and the properties of emulsions are included. In the third group the subject of photographic sensitivity is considered. Chemical sensitization in emulsions, and dye sensitization are included. Two papers by W. West and by B. N. Carrol and W. West on dye sensitizer action are worthy of special consideration. In the fourth group are found papers on latent image formation, in which group falls solarization and the Herschel effect, distribution of latent image through the grains, etc. The paper by Mitchell mentioned above is in this group. The last group of papers, constituting almost one-third of the total number, is concerned with processing and properties of the nuclear track emulsions.

The title of this book would lead one to believe that it is a source of information on the fundamental mechanism of the formation of the photographic image. This in turn, to this

reviewer at least, necessarily means that collected in this book should be a group of review papers covering the various aspects of the latent image, etc. Instead, the papers given in this book are completely experimental, and as such, are quite restricted in scope. In no sense do they give an over-all picture of our present ideas about the fundamental mechanism of photographic sensitivity. In this sense, this reviewer was disappointed in the book. On the other hand, as a collection of experimental papers it is quite good. The papers are about what one would expect to find in an issue of one of our leading scientific journals in that there is about the same variety of subject matter, the same lack of close relationship among the papers, the same sprinkling of good and bad papers. In one sense, the collection is perhaps somewhat superior to what is usually found in a given journal issue since it contains contributions from the leading workers in the photographic field. The objection then boils down to one of title and price, which seems a little high to pay for a single issue of a scientific journal. In other respects the book is excellent. It is printed on good paper and the format, as is customary in good English books, is excellent. It is a useful collection of experimental papers in the field of photographic sensitivity, but not an expecially significant one.

ALAN D. FRANKLIN

MICROBIAL DECOMPOSITION OF CELLULOSE, by R. G. H. Siu. 531 pages, illustrations, diagrams. 16 × 24 cm. New York, Reinhold Publishing Company, 1951. Price, $10.00.

The problem of microbial decomposition of cellulose is discussed as to probable mechanisms, organisms responsible, possible means of prevention, and the losses, economic and otherwise, involved. A truly imposing number of references has been compiled by the author. The book is profusely illustrated with photographs, diagrams, and graphs, and much of the material is presented in concise tables which certainly contribute a good deal to the usefulness of the book.

The book is written primarily for persons interested in a rather narrow, but very important, field of microbial action. However, the information presented should interest practically all students of mycology and bacteriology. The sections dealing with causal organisms and the mechanism of degradation of cellulose, particularly, should be of general interest.

Aside from the practical facts which are evident in great abundance throughout the text, the most important feature of the book is a frank discussion of the limitations of the available knowledge in the field. Care is taken to point out discrepancies in data and possible explanations, if any, for these discrepancies. This discussion should be of particular value to the researcher in that the confusing array of information is compiled and evaluated, thus allowing the worker to plan future study in the areas which should be most fruitful.

WARREN R. STINEBRING

SCIENCE IN PROGRESS, edited by George A. Baitsell. 512 pages, diagrams and illustrations, 16 × 25 cm. New Haven, Yale University Press, 1951. Price, $6.00.

The seventh in the series "Science in Progress" is now available. This series, edited by George A. Baitsell, Professor of Biology at Yale, contains the most significant advances in science to date contributed by the men responsible for these investigations. The book consists of chapters originally presented as material prepared for the Sigma Xi National Lectureships.

The book opens with the fascinating story of the beginning of life as seen by the development of a chick heart and the story behind the micro-moving picture studies are described by the author, Dr. Bradley M. Patten.

The other chapters are devoted to "The Reproductive Cycle of the Rhesus Monkey," "Human Infancy and the Embryology of Behavior," "Radiation Damage to the Genetic Material," "Genes," the structure of biological materials, a review of the elementary particles of physics, a comparison of atomic and solar energy, atomic structures, a mathematical discussion of the theory of Braids, the history of South American animals and finally the chemistry of polymers.

Of the twelve authors, ten are members of the National Academy of Sciences and two of the ten—Drs. H. J. Muller and Carl D. Anderson—are Nobel Prize winners. The stature of

these science pioneers in their respective fields insures the serious consideration of the works presented even though the conclusions may be at variance to the accepted opinions today.

This book is a grandstand seat for the reader interested in the dynamic procession of science.

I. M. LEVITT

ADVANCED DYNAMICS, by E. Howard Smart. Volumes 1 and 2, diagrams, 15 × 23 cm. London, MacMillan and Co., 1951; distributed in U. S. by The Macmillan Co., New York. Price, $12.00.

These works were prepared from the lectures given by the author to advanced students preparing for both the General and Special Degrees in the University of London. They furnish a very thorough account of dynamics developed in such a manner as to be particularly suitable for university graduate students who are interested in both pure and applied mathematics. The contents of the volumes are not well outlined. Instead, the treatments are arranged in a series of one-topic paragraphs much as in a lecturer's notebook. Because of this form, the books are most suitable as a text rather than as a reference. Nevertheless, because of their comprehensiveness, they will also be valuable as a reference for any who wish to view the subject from the mathematics standpoint, or wish to apply the subject in the fields of engineering, aeronautics, physics, etc.

Volume I, "Dynamics of a Particle," includes a thorough treatment of the fundamental dynamic principles. It gives a complete and interesting discussion of rectilinear motion with acceleration directed toward and away from a fixed point; oscillating particles; simple harmonic motion with unresisted and damped vibration; and impulsive forces applied to problems of impact.

Volume II, "Dynamics of a Solid Body," embraces a complete discussion of d'Alembert's principle and the equations of motion in two and three dimensions; Lagrange's equations and generalized coordinates; the motion of a top; Hamilton's equations and general theorems on impulses. Frequent historical notes are given on those who laid the foundations of mathematical dynamics.

No effort has been spared to apply the calculus, wherever possible, to establish principles as well as to solve problems. In this respect the author's methods are sound and excellent continuity is maintained.

The writer recommends these volumes highly for their comprehensiveness, their excellent examples, and their clear mathematical approach to problems in applied dynamics.

RICHARD J. SHUCK

PRACTICAL PHOTOGRAMMETRY, by H. Oakley Sharp. 229 pages, illustrations, 13 × 21 cm. New York, The Macmillan Co., 1951. Price, $3.75.

Photogrammetry is the science, or perhaps the art, of preparing topographic maps from either aerial or ground photographs. This reviewer's only justification for writing this review is the fact that he attended the Air Force School of Photogrammetry at Lowry Field, Denver, Colorado, during the war. At that time, this book would have been extremely useful, and so it is recommended to those who want a practical handbook in this subject. It is basically a textbook, with enough detail to act as a reference, and yet broad enough to be a refresher for the practical engineer involved in making the maps.

The book is divided into two sections, labeled "Theory" and "Practical Application." The "Theory" section is concerned with the geometry and mathematics underlying the process of transferring information from the photographs to the map. It includes the geometry of the camera, of this transfer process, of stereoscopic vision, and a brief description of methods of control and of plotting of the actual maps. By control, the photogrammetist means the plotting of ground points whose positions are determined by standard surveying methods, and, being identified in the photographs, serve to tie them down to definite positions on the map. This section also contains a description of some of the instruments that are used, such as aerial cameras, enlargers, stereoscopes, etc. The second section labeled "Practical Applica-

tion," contains detailed instructions for the various operations involved in map-making. It includes sample problems, and a section on the planning of aerial surveys.

The general nature of this book is that of a very practical description of its subject, and should be most useful to the man who actually has to make the maps. In this capacity it should be an extremely valuable little book.

ALAN D. FRANKLIN

THE DESIGN OF SWITCHING CIRCUITS, by William Keister, Alistair E. Ritchie and Seth H. Washburn. 556 pages, diagrams, 16 × 24 cm. New York, D. van Nostrand Co., Inc., 1951. Price, $6.00.

"The Design of Switching Circuits" is an interesting and well written text on relays and other switching circuits. The first half of the book gives the general principles for designing basic circuits utilizing relays, switches, and electronic elements. Chapters 11 through 21 indicate the technique of designing uni-functional circuits and present a wide variety of examples of different types of such circuits. The last three chapters outline the method of designing multi-functional circuits or complete units. The chapter on logic is of real value to the reader because of the simple explanations, the summary tables of formulas and the numerous illustrative examples.

Engineers and technicians desiring a better knowledge of switching circuits in general and relay circuits and relays in particular will find this book a suitable text for home study or general reading. The numerous illustrative examples are supplemented at the end of each chapter with a variety of interesting problems.

The authors draw heavily on their experience with telephone circuits and the large Bell Relay Computer circuits for much of their illustrative material. They emphasize the reliability of relay switching apparatus as contrasted with electronic apparatus. The application of the basic principles to electronic circuits is discussed in general terms and illustrated with some specific examples.

This is an excellent theoretical and practical text.

E. A. MECHLER

LANGUAGE AND COMMUNICATION, by George A. Miller. 298 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $5.00.

This book was written as a text for an advanced undergraduate or graduate course in the psychology of language and communication. The particular psychological orientation which the author maintains is that of behaviorism; this is to say a successful attempt is made to describe and interpret the observable data of language and communication in objective terms. Much of the text deals with aspects of communication and language, which although certainly of cultural interest to the engineer, are not of direct practical value. This fact is not a shortcoming of the text but rather a consequence of the author's purpose in writing it.

On the other hand, there are several chapters which would be of real value to the engineer designing systems or equipment for use by human operators. Chapter 3 provides a well written introduction to the facts known about the perception of speech. The effects on speech perception of such things as masking, filtering the sound spectrum, clipping, speed of talking, and so forth are discussed. Chapters 4 and 5 are quite interesting from the engineer's viewpoint in that they describe methods for the statistical study of language, following Shannon, and present a simple discussion of information theory as it applies to the use of symbols.

E. S. KRENDEL

INTRODUCTION TO NUMBER THEORY, by Trygve Nagell. 309 pages, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $5.50.

Professor Nagell has written a very readable account of the subject. Although the book is introductory it contains not only the fundamental results in number theory, but also some results which are still part of current research, and thus it should be an excellent background for the student desiring to do further study in this field. However, the book is not written for a reader with a great deal of mathematical sophistication, and a very modest background in algebra should suffice for all but a few sections of the book.

There are eight chapters. The first deals with divisibility, while chapter two discusses the distribution of primes. A long discussion of the theory of congruence is contained in chapter three. Chapter four is devoted to quadratic residues, and roots of unity are treated in chapter five. Diophantine equations of the second and higher degree form the subject matter for chapters six and seven, the linear diophantine equations having already been covered in chapter one. The last chapter covers an elementary proof of the prime number theorem given by Atle Selberg (1948).

The subject is developed rigorously. However, the material is motivated by a historical discussion of the development of the subject and difficult theorems are followed by illustrative examples. The book contains many exercises, but these are far from trivial and are designed to supplement the material contained in the text.

H. L. PLATZER

PHYSICAL CHEMISTRY OF LUBRICATING OILS, by A. Bondi. 380 pages, illustrations and diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1951. Price, $10.00.

In order to aid the understanding and interpretation of scattered physicochemical data from research on lubricating oils, the author attempts to erect a framework of theory and chemical structure *vs.* physical property correlation into which to fit available data. The author points out that the theoretical background presented is in some cases elementary, but he feels it has been neglected by experimenters, a statement with which this reviewer agrees. Another aim of the author is to present the status of the field rather than a history of its development. It is believed that this objective has been fairly well accomplished. Chapter headings are as follows: Pressure-Volume-Temperature Properties, Rheology, Surface Phenomena, Optical and Electrical Properties, Hydrocarbon Type Analysis of Lubricating Oils, Phase Equilibria, Reaction Kinetics, and Synthetic Lubricants. The references cited indicate that the survey in some cases, for example, the material on reaction kinetics, extends only to 1947. Other chapters concerning subjects to which the author has personally contributed are up-to-date.

On the whole, the book is very readable, an accomplishment when it is considered that the material comes from many sources. The arrangement and printing are good. The book is recommended for petroleum chemists and engineers in particular, and for those specialists in some part of the field who wish to obtain a general over-all picture.

W. E. SCOTT

THE CHEMICAL TECHNOLOGY OF DYEING AND PRINTING, by Louis Diserens. Volume 2, translated and revised from the second German edition by Paul Wengraf and Herman P. Baumann. 446 pages, 16 × 24 cm. New York, Reinhold Publishing Corp., 1951. Price, $12.00.

This very practical book for workers in the dyeing and printing industry is a translation from the German. It details the application to textiles of substantive, basic, and acid dyestuffs, aniline black, and dyestuffs for cellulose acetate, nylon, and other synthetic fabrics, as well as protein and glass fibres.

The chapters on the various classes of dyestuffs are replete with formulas and detailed procedures and amply referenced. The difficulties commonly encountered in the application of specific dyestuffs are discussed, and the proper precautions listed.

A brief discussion of the theory and chemistry involved in the printing and dyeing with several classes of dyestuffs should be of interest to the workers in the industry. Tables of auxiliaries for dyestuffs are included at the end of each chapter listing the name, manufacturer, chemistry, and dyeing characteristics of each of the compounds.

A chapter is devoted to the history, physical and chemical properties, and the special problems created by the development of the new synthetic fibres, such as Vinyon, Orlon, Saran, Terylene, Nylon, and others. The use of resin emulsions to achieve pigment fixation in printing on textiles is discussed in detail.

The bibliography is voluminous, and the references extensive.

S. N. MUCHNICK

BOOK NOTES

AN ELEMENTARY TEXT IN HYDRAULICS AND FLUID MECHANICS, by Ralph W. Powell. 316 pages, illustrations and diagrams, 15 × 22 cm. New York, The Macmillan Co., 1951. Price, $4.75.

This is a revised text based on the writer's "Mechanics of Liquids." The material is organized to give the student a foundation in fluid mechanics and current practice. Hydrostatics, fundamentals of fluid flow, orifices, tubes, nozzles and weirs, pipe flow, the effects of viscosity, hydraulic models, properties of liquids, and dimensional analysis are covered.

TV AND ELECTRONICS AS A CAREER, by Ira Kamen and Richard H. Dorf. 326 pages, illustrations, 14 × 21 cm. New York, John Rider Publisher, Inc., 1951. Price, $4.95.

The facts about the electronics industry are presented to answer the questions of persons interested in entering the field. Radio and television broadcasting, communications, manufacturing, engineering, distribution, and sales and servicing are presented by professionals in each field. It is a very readable and informative book.

A BIBLIOGRAPHY ON "THE EFFECTS OF X-RAY ON BACTERIA," by Radcliffe F. Robinson, Michael D. Phillips and Margaret G. Nagelsen. 18 pages, 23 × 30 cm. Columbus, Battelle Memorial Institute, 1951. Paper cover.

This bibliography has been prepared by members of the Battelle research staff. Recent developments in radiation sterilization make this a timely contribution to the radiation research worker's library. Arranged by years (from 1896 through 1951), the 178 entries are widely spaced on the pages, facilitating easy use. A limited edition is being distributed on request to Battelle.

THE ANNUAL REPORTS ON THE PROGRESS OF CHEMISTRY for 1950. Vol. 47. 490 pages, 14 × 22 cm. London, The Chemical Society, 1951. Price, 25s.

A thorough report on all the fields of chemistry for the years 1949 and 1950. The report is most useful in the physical and physicochemical methods section of analytical chemistry. The section covering crystallography includes papers published over a period of four years 1947–1950, inclusive. An excellent and complete review on correlation of experiment and theory dealing with bond distances in crystallographic computing methods. Antibiotics and neutron crystallography are included.

ANNUAL REPORT ON THE PROGRESS OF CHEMISTRY, Cumulative Index, Vols. 1–46, 1904–1949. 286 pages, 14 × 22 cm. London, The Chemical Society, 1951. Price, 25s.

The cumulative index is a subject index only, and is limited in its scope. Forty-two years of work are presented in 286 pages. The index will be most useful for those persons working where a complete file of the "Annual Reports on the Progress of Chemistry" is available.

STEEL SERVES THE NATION 1901–1951, by Douglas A. Fisher. 225 pages, illustrations, 23 × 30 cm. New York, United States Steel Corp., 1951.

The fifty year story of United States Steel is portrayed herein, including the important role which steel has played in the spectacular economic growth of United States during the last half-century. This Golden Anniversary Book of U. S. Steel presents a factual summary of the principal achievements and services of the corporation since its formation. Beautifully bound and filled with excellent photographs, some in color, the book will serve as a source of information not only of United States Steel Corp., but of the steel industry in general.

DATA BOOK FOR CIVIL ENGINEERS, by Elwyn E. Seelye. Second edition. Volume I, Design; Volume 2, Specifications and Costs. 24 × 30 cm. New York, John Wiley & Sons, Inc., 1951. Price: Vol. 1, $10.00; Vol. 2, $13.00.

The first two volumes of this monumental work have been brought up to date. Still adhering to its original purpose—to provide a civil engineer with enough data to design any civil engineering work without reference to other sources—the second edition has been completely revised and several additions have been made. In Volume 1, the new material includes fuller treatment of soil mechanics; airfield design data; design of swimming pools; industrial waste treatment; added equipment for dams; and new turnpike standards. Also, a wider range of "push-button" design data has been included. Volume 2 has been brought up to date with the addition of specifications for elements of industrial buildings, of swimming pools and cost information on such items as ski tows, drive-in theaters, etc.

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exists today because of the faith and generosity of the men and women who for 127 years have given time and money to its support.

The Institute welcomes financial gifts and bequests and hopes that all those who desire to perpetuate its work will make the The Institute one of their beneficiaries.

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Ask Astia for technical information and bibliographic services. This new Armed Services Technical Information Agency has been organized by the Defense Department to collect, catalog, and store technical information generated by or for the military departments, and to provide bibliographic service on scientific and technical reports for military agencies and their contractors, and other authorized agencies. First task of ASTIA will be integration of technical information services performed by CADO (Central Air Documents Office) and Naval Research Section of the Library of Congress. Director is Leslie E. Neville, former Director of Public Relations for Curtiss-Wright. Address: ASTIA, HQ, U. S. Air Force, Room 5D518, Pentagon.

New Tungsten Machining Technique Developed.—A new technique for drilling, grinding, turning, milling, threading, and tapping tungsten metal using ordinary metal working machines has been developed by Philips Laboratories, Inc., Irvington, N. Y.

According to an announcement by Dr. O. S. Duffendack, President, this new technique permits the machining of tungsten metal to tolerances comparable to those normally achieved with steel or brass. For instance, tungsten tubing has been fabricated with an outer diameter of .066 in. ± .0005 and an inner diameter of .060 in. ± .001. An 0–80 tungsten screw $\frac{11}{16}$ in. long has been made with a .025 in. hole drilled through its entire length.

The Laboratory is already using this new method in manufacturing its newly developed tungsten L-cathodes for long lived electron tubes. Additional applications are expected to be found in electron tube manufacture and in the fabrication of parts of other devices which must operate at exceedingly high temperatures in a vacuum or in reducing or inert atmospheres.

The new machining technique was developed by Dr. Roberto Levi, a research chemist on the Laboratory staff. Up to the present it has not been possible to make complicated shapes of tungsten, due to its extreme hardness and brittleness. Rod, wire, and sheets are its present common forms.

Stereo-photogrammetric Mapping Instrument.—The Kelsh Plotter, a relatively new stereo-photogrammetric mapping instrument, has been tested and evaluated for military application at Fort Belvoir's Engineer Research and Development Laboratories.

The instrument was originally designed by Harry Kelsh who was then with the Soil Conservation Service. It was further developed by the Geological Survey for use in its mapping program.

The Kelsh Plotter, in principle, is similar to the Multiplex, a standard stereo-photogrammetric mapping instrument of the Corps of Engineers. Both instruments project a three-dimensional, measurable image from a stereo-pair of aerial photographs, from which a topographic map may be drawn. The Multiplex requires auxiliary equipment to correct for aerial camera lens distortion present in the aerial photograph, whereas the Kelsh Plotter has a correction device built into the instrument. Moreover, the three-dimensional image projected by the Kelsh Plotter is more highly resolved and at a greater magnification than that projected by the Multiplex.

The commercial model tested by the Topographic Engineering Department at ERDL, although lacking certain desirable features, revealed a basic design favorable to military mapping applications. Accordingly, modification and further development of the instrument are being actively pursued.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

A Procedure for the Preparation of 2-Desoxy-D-glucose.—FRANCIS B. CRAMER. Moderate quantities of 2-desoxy-D-glucose have been repeatedly prepared for use in experimental work being carried out at this laboratory. The procedure evolved is a modification of the original glucal method of Bergmann (1). Although the method has by no means been brought to a state of perfection, it is effective and reliable.

The chief advantage of the procedure is that it avoids unnecessary isolation of intermediates, which is wasteful and time-consuming. Glucose is acetylated rapidly at high temperature with a trace of sulfuric acid catalyst. Acetobromo glucose is prepared immediately by the addition of hydrogen bromide to the acetylation mixture. The resulting solution is added directly to a reducing mixture containing sodium acetate to neutralize excess hydrogen bromide (2). At this point the reaction product, glucal triacetate, is isolated and purified. The glucal triacetate is rapidly de-esterified by boiling methanol using a sodium methylate catalyst (3), the solvent is removed, and the resulting glucal is hydrated by standing overnight with very dilute sulfuric acid at room temperature.

The successful use of 0.035 normal sulfuric acid for the conversion of glucal to desoxyglucose is scarcely compatible with the generally accepted theory that sulfuric acid adds to the double bond of glucal and is subsequently removed by hydrolysis (4). A more probable mechanism would be the proton-catalyzed opening of the oxygen bridge of the glucal, resulting in a vinyl alcohol structure which would instantly rearrange to the aldehyde form.

Difficulty is sometimes experienced in obtaining the first seed crystals of glucal triacetate and desoxyglucose. Glucal triacetate should be purified by several vacuum distillations, retaining the middle fraction. Scratching and chilling in a dry-ice bath should induce crystallization. Desoxyglucose seed crystals are best obtained through purification by the method of Bergmann (1), followed by repeated trituration of the syrup with absolute ether and drying in a vacuum over solid sodium hydroxide.

The procedure as presented is not entirely adaptable to the preparation of the 2-desoxypentoses. The acetobromo pentoses may be prepared as described but extremely low yields of the glycal diacetates are obtained by direct addition of the solution to the reducing mixture. The glycal diacetates must not be boiled with sodium methylate, but may be de-esterified by the method of Isbell (5). Hydration of the glycals should be carried out at a low temperature.

PREPARATION OF GLUCOSE PENTAACETATE

Weigh 120 gm. *anhydrous* D-glucose. Break up all lumps by grinding in a mortar. Put into a one-liter 3-neck flask fitted with a mechanical stirrer and a thermometer. Add 350 ml. acetic anhydride (E. K. practical is suitable).

Prepare a catalyst by dissolving ½ ml. concentrated sulfuric acid in 5 ml. glacial acetic acid. Add about .1 ml. of the catalyst solution to the reaction mixture; then add more catalyst solution in small quantities until esterification has begun as indicated by a regular rise in temperature. Hold the temperature at 80° C. or lower by the application of a cold water bath as needed. (The speed of the reaction and the amount of catalyst required will depend on the fineness of the crystals of sugar and on the amount and kind of impurities present in the sugar and the anhydride. Stand back to avoid acid burns if the temperature gets out of hand and the reaction boils over.) The reaction should be essentially complete in ½ to 1 hr. This mixture may stand at room temperature indefinitely.

PREPARATION OF ACETOBROMO GLUCOSE

Tare the flask containing the acetic acid solution of glucose pentaacetate. Stir very vigorously and introduce (through a glass tube reaching to the bottom of the flask) gaseous hydrogen bromide from a cylinder of the liquified gas. (Four-pound cylinder from Matheson, East Rutherford, N. J. is convenient.) Much heat is evolved. Hold the reaction mixture at about 20° C. by a dry ice-acetone bath at −30° C. Pass in hydrogen bromide until 200 gm. have been dissolved. This can be accomplished in about 15 min. Allow to stand (not over 2 hr.) at room temperature while preparing to carry out the next reaction.

PREPARATION OF GLUCAL TRIACETATE

Use a 3-liter, 2- or 3-necked flask provided with a thermometer, a 500-ml. dropping funnel, and a mechanical stirrer (a large paddle driven at slow speed is preferable to a small propeller driven at high speed). Put in 180 gm. fresh zinc dust, 175 gm. sodium acetate trihydrate, and 305 ml. distilled water. Stir until crystals are dissolved. Cool to about 0° C. (dry ice-acetone bath). Dissolve 18 gm. copper sulfate pentahydrate in 95 ml. water and add about two-thirds of this to the mixture, in a slow stream while stirring well. Again cool to 0° C. Add 345 ml. glacial acetic acid. Cool to 0° C.

Place one-half of the solution of acetobromo glucose in the dropping funnel and add it to the well-stirred reducing mixture in a thin stream during 20–30 min. The reaction mixture must be maintained at 0° to 0.5° C. (dry ice bath at −10° to −15° C.).

Now prepare a solution of 175 gm. sodium acetate in 225 ml. water, and mix with the rest of the copper sulfate solution. Put 50 gm. fresh

zinc dust into the reducing mixture and add the sodium acetate-copper sulfate solution with good stirring. Cool to 0° C. and add the second half of the acetobromo glucose solution as before. When the solution has all been added stir the reaction mixture for about 15 min. before proceeding. (The reaction mixture may safely be left standing in the cold for an hour either before or after filtering but should not be left overnight at this stage.)

Filter the reaction mixture on a charcoal-coated filter (filtration is slow and two 130 mm. Buchner funnels can be used to advantage). Wash residue with distilled water. The residual zinc is pyrophoric and must not be sucked dry on the filter. The filtrate is diluted with about 1 liter distilled water (including washings) and extracted with benzene (a 4-liter separation funnel is convenient). Extract with two 350 ml. and one 100 ml. portions of benzene. Each extract is washed twice with about 150 ml. water and then 150 ml. saturated sodium bicarbonate solution. Dry the combined extracts with anhydrous sodium sulfate and filter. (The solution may now be left overnight if necessary.) Evaporate the benzene in vacuum. Take up the residual thin syrup in 100 ml. 95 per cent alcohol.

Evaporate in vacuum again to get rid of the last of the benzene. Dissolve the residual syrup in 200 ml. 95 per cent alcohol, chill in the refrigerator (stainless steel beaker preferred), seed with crystalline glucal triacetate and stir until thick. Leave in refrigerator overnight.

Break up the crystalline cake with a heavy glass rod, filter the slurry with suction, and press out well with a flat stopper. Wash quickly with 50 ml. cold alcohol. Suck as dry as possible on the filter. Break up cake and dry thoroughly in the open air at room temperature. Yield, about 125 gm. The dried product may be used in the next step without further purification. Pure glucal triacetate melts at 55° C. and has a specific rotation of minus 13.0° in alcohol (6). It may be distilled at 2 mm. pressure with a boiling point of about 125° C.

It is convenient to prepare and dry two batches of glucal triacetate before proceeding to the next step.

PREPARATION OF GLUCAL

Dissolve one batch of glucal triacetate (about 125 gm.) in 500 ml. absolute methanol in a liter flask. Heat the solution almost to boiling on a steam bath. Add 35 ml. of 0.1 *N* sodium methylate in methanol. Boil vigorously for 10 min. Test a few drops of the solution in phenolphthalein and water. If the solution is not still alkaline, add more sodium methylate and boil 10 min. again. When the solution tests alkaline after boiling, add a generous lump of dry ice and 100 ml. water. Evaporate the methanol in vacuum. During the evaporation, de-esterify the second batch of glucal triacetate as above. Combine with the first batch and evaporate in vacuum to a thin syrup.

PREPARATION OF DESOXYGLUCOSE

Take up the glucal syrup in about 700 ml. distilled water. Add 40 ml. of approximately *N* sulfuric acid. Dilute to approximately 1000 ml. Allow to stand at room temperature overnight (16–18 hr.).

Neutralize the sulfuric acid with an excess of reagent grade barium carbonate. Warm to about 40° C. and shake occasionally for 1 hr. Add a generous portion of decolorizing carbon and filter through a carbon-coated filter paper. Evaporate in vacuum to a thin syrup. Avoid high temperatures. During the evaporation, solid barium carbonate will form due to the decomposition of barium bicarbonate in solution. Take up the residue in 200 ml. of dry isopropyl alcohol. Filter through a charcoal-coated filter. Again evaporate the filtrate in vacuum to get rid of water. It is usually advisable to take up the residue in 100 ml. of isopropyl alcohol and evaporate again to insure complete dehydration. Take up the final residue in 150 ml. isopropyl alcohol and seed with desoxyglucose. Stir frequently during the first half hour. Allow to crystallize overnight at room temperature to reach mutarotational equilibrium. Filter with suction and wash with 50 ml. isopropyl alcohol. Dry in open air. Yield, about 50 gm. The product will be of egg-shell color. For purification, dissolve in about 150 ml. distilled water and filter with a generous amount of decolorizing charcoal. Evaporate the colorless filtrate in vacuum and dehydrate by evaporation with isopropyl alcohol as before. Crystallize and wash with isopropyl alcohol. Yield, about 45 gm.

The desoxyglucose usually crystallizes as a mixture of the alpha and beta anomeric forms, and so does not have a definite melting point. Mutarotation in water is very rapid and reaches an equilibrium value of plus 46.6° in a few hours (1) (1 per cent solution in water).

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CURRENT TOPICS

Breeder Reactor Generates Power.—Small amounts of electric power have been produced from heat energy released in the operation of the experimental breeder reactor, recently completed at the National Reactor Testing Station in Idaho, it was announced recently by A. Tammaro, Manager of the Chicago Operations Office of the Atomic Energy Commission.

In a trial run, electrical power of more than 100 kilowatts was generated and used to operate the pumps and other reactor equipment and to provide light and electrical facilities for the building that houses it. Test operations will be resumed after further adjustments of the reactor system.

The heat energy generated was removed from the reactor by a liquid metal at a temperature high enough to generate steam to drive the turbine.

The principal function of the breeder reactor is the long range goal of converting nonfissionable material into fissionable material more rapidly than nuclear fuel is consumed, a process that would contribute to expansion of the current atomic weapons program.

The power generation phase is incidental but is being carried out to secure experimental information on the handling of liquid metals at high temperatures under radioactive conditions and on the extraction of heat from a reactor in a useful manner. The system at the breeder reactor can never generate large amounts of electrical power but it does provide a useful tool for carrying out such experimental studies.

No comparisons should be made of the cost of producing electric power from this reactor with power from conventional sources. Cost was not an essential factor in the power phase of the Idaho reactor and the experiment is in no way intended to establish the feasibility of producing electrical power economically from nuclear sources. The technical information gained, however, may be useful in the design of future reactors aimed at generating electricity at a competitive cost.

The Rotary Draw Former.—Faster, longer flying and safer jet planes are closer to mass production, thanks to the Rotary Draw Former, developed by the Cyril Bath Machinery Co., of Cleveland. This tool, developed from experience with the Bath Universal Contour Former, has proved particularly useful in stretch-forming Haynes-Stellite, and various vanadium alloys used in the afterburners and combustion chambers of jet engines. It also can produce a number of other jet engine parts which have not been readily produced by other methods.

The Rotary Draw Former consists of a power driven rotary steel table, similar to a Vertical Boring mill, and a horizontal bed on which is mounted a hydraulically movable carriage. On the carriage is mounted a swivellable ram, like a large shaper—operated by a hydraulic cylinder and the whole element mounted on roller bearings.

On the ram is adjustably mounted one of two stretch jaws. The other jaw is mounted on the rotating table and the work is stretched between them.

The work previously is shaped roughly to the required cross section as are also the gripping jaws. The metal is then stretched between the jaws, and this pressure is automatically held constant during the forming operation. The table is then revolved. This causes the metal to be stretched or drawn over the outer diameters of the shape mounted upon the table, the amount of draw becoming progressively less toward the inner diameter where very little elongation may be required.

Simultaneously, to assist the process and to assure accuracy in cross section, a side head applies hydraulic pressure through a hard bronze shoe, wiping the metal onto the revolving die as the table revolves.

Rings up to 48 in. in diameter have been drawn with a ten thousandths tolerance for variation in cross section of circumference.

In the final operation the rings are held in a welding fixture and automatically welded with atomic hydrogen, the cooling being so rapid that distortion in the finished ring is held to a minimum. No re-stretch operation is required and, indeed, in these tough metals restretching is seldom satisfactory.

The Rotary Draw Former will produce elliptical or rectangular shapes equally well. Where complete circular or closed shapes are required, the work is drawn on a spiral die and the ends are sprung together for the final welding operation.

The same machine will also produce long sweeping open curved shapes, shapes containing reverse bends, or curves in two planes. It will form aluminum or bronze, and alloy extrusions, and will vary the angularity of the extrusion, as in wing attach angles or tank straps, while forming the work. It will form round, square, or other hollow tube shapes. Sheets of all sorts and sizes can be handled by changing to suitable jaws.

It will work equally well in the toughest steels, or in very light aluminum extrusions, the latter by reason of the hydraulically balanced ram and the roller mounted stretch ram and cylinder which respond to very slight pressures or resistance.

The dies in use for rotary work consist of a turned cast iron base on which is mounted a strip of steel so shaped as to give the required cross section. This ring is either turned up to size and split, and welded in place spirally, or it is planed and one end welded to the die. The Rotary Draw Former ram then wipes the steel bar spirally around the cast iron base and it is so welded into position.

Dies so made cost much less to produce than press dies and will make shapes virtually unobtainable by any other means. The metal, prestretched and applied to the die while in this condition, gives very good product in the very tough materials needed in jet engine manufacture and at a minimum cost and loss from scrap.

The Rotary Draw Formers are produced in six sizes: 12½, 25, 35, 50, 75, and 100 tons.

The Cyril Bath Company's standard Contour Former can be converted to a Rotary Draw Former by the addition of appropriately designed self-contained supplementary parts.

Low-Cost Electric Locomotive.—A new low-cost electric locomotive which can handle main-line freight traffic on nearly 90 per cent of America's electrified

lines is the electrical industry's most significant 1951 contribution to the railroads.

In announcing the new General Electric a-c. locomotive designed for quantity production, G. W. Wilson, manager of G. E.'s Locomotive and Car Equipment Department, stressed the railroads' increased use of electrical equipment to help move goods and people efficiently.

"Heavier trains and higher speeds characterize modern railroading," Mr. Wilson said, "and versatile, more powerful locomotives are needed to supplement or replace those now operating on existing electrified lines."

The G-E executive pointed out that rising transportation costs have forced the railroad industry to seek a relatively inexpensive standard electric locomotive which can operate efficiently at a low maintenance cost.

General Electric's contribution is a single-phase, a-c. locomotive which is designed for mass production. The design is based on fundamentals proved by 50 years' experience in electric locomotive manufacture. The streamlined cab carried on two 2-axle swivel trucks is essentially similar to the diesel-electric freight locomotives now popular on American railroads. All axles are powered by a-c. commutator motors. Dynamic braking is used—the first time it has been applied to this type of locomotive in the United States.

Mr. Wilson said that the new units operate from 11,000-volt, 25-cycle, single-phase, a-c. trolley wire, used by the majority of electric locomotives in this country.

Each unit has a continuous rating of 2500 hp., and can develop as much as 5000 hp. for short periods. As many as four units can be operated in multiple under the control of one engineer.

TV and Radio Coverage of 1952 Election Campaign.—Nationwide television and radio coverage of the 1952 presidential election campaign will be sponsored by Westinghouse Electric Corporation over the Columbia Broadcasting System TV and radio networks, it was recently announced by J. M. McKibbin, Westinghouse vice-president in charge of consumer products.

Westinghouse-sponsored CBS networks will cover the hundred largest population areas in the United States. It will be the first time the Republican and Democratic conventions, as well as the general elections, have been televised on a national hookup.

The most complete program package ever developed for a presidential campaign, it will include full coverage of both political party conventions in Chicago, a 13-week non-partisan "Get-Out-the Vote" campaign, and election returns the night of the election on November 4.

The 13-week "Get-Out-the-Vote" campaign will start in August and continue until the night before the election. It will offer equal time to both major parties to present their campaign issues through their candidates and other leading political figures.

"Westinghouse is bringing the conventions, the election, the candidates, and the campaign issues to the American people to stimulate maximum interest in the world's greatest demonstration of democracy at its best," Mr. McKibbin said. "We have planned this program to help produce the biggest turnout of well-informed voters in the history of American elections."

Radiologistics.—The use of a new scientific term, "radiologistics," was proposed by Myron Weiss, New York science editor, to the American Association for the Advancement of Science, as that largest group of scientists in the world held their convention in Philadelphia recently.

"Radiologistics," Science Editor Weiss points out, "describes the applications of the knowledge of radiations and other sub-atomic phenomena in the examination, identification, alteration and creation of primary and organized matter—by means of natural elements (radium, etc.), artificial elements (plutonium, etc.), altered elements (carbon, cobalt, etc.), and universal sub-atomic phenomena (light, cosmic rays, mesons, etc.)—in research, industry, medicine, warfare, etc."

New Synthetic Base for Photographic Film.—An entirely new synthetic base for photographic film has been developed by du Pont research. Preliminary tests show that it is several times tougher and has much greater dimensional stability than any of the present types of film base. It is classed as a safety base, and is less flammable than present safety bases.

The new material, technically known as a polyester, is chemically related to "Dacron" polyester fiber, the newest of the company's synthetic textile fibers. Both are condensation polymers made from ethylene glycol and terephthalic acid.

Du Pont's Photo Products Department is currently producing polyester base in laboratory scale equipment and will shortly start up a new pilot plant at its Parlin, N. J., laboratory.

Polyester base is exceptionally tough, a characteristic that is particularly advantageous for motion picture use. It has twice the tear resistance of the standard acetate or nitrate base film, and can be run through a projector from three to four times as long before perforations show appreciable wear. Because of its toughness and greater stiffness, it is believed that polyester film may make it possible to reduce the thickness of motion picture film by at least 20 per cent, with a resulting greater footage per reel, and accompanying savings in processing and handling.

The dimensional stability of the new base offers important advantages, particularly in the motion picture industry and graphic arts, where close tolerances are essential. Polyester base keeps its shape to a remarkable extent even under the most extreme conditions, and such microscopic changes as do occur are much smaller than in existing films. Another important characteristic of the new base, its lack of brittleness at low temperatures, combined with its dimensional stability, makes it especially well suited for aerial mapping films which must frequently withstand low temperatures and humidities.

Five years of research and an investment of more than one and a quarter million dollars have gone already into the development of this new film base, the company said. It added that the decision as to large-scale production would await the results of the evaluation tests now being conducted in cooperation with the Motion Picture Research Council and others having special interest in the unique properties of the new material. If it meets all tests satisfactorily, more than two years will be needed to design and complete large-scale manufacturing facilities, the company emphasized.

Portable Analog Field Plotter.—A new portable Analog Field Plotter, a versatile tool for rapid solution of complex two dimensional field problems, has been announced by the Special Products Division of the General Electric Company.

The new G-E product sets up electrical field patterns in a thin conducting-paper surface on the plotting board. Analogy between the electric field in the paper and the related field problems—such as might exist in electrostatics, electromagnetics, thermal and fluid flow, etc.—allows an easy solution to a broad range of difficult problems.

According to engineers of the G-E General Engineering Laboratory who developed it, the Analog Field Plotter will be a valuable tool in the industrial and classroom study of electrostatic field problems, determination of magnetic flux distribution and intensity, and charting of current flow and temperature distribution in conductors of varying cross-sections and configurations.

Since the new plotter can be used for the study of air flow patterns, as well as electrical fields, fluid flow and temperature distribution, it is expected to have extensive use in the aircraft industry in the study of air-flow patterns around air foil shapes in wing and propeller designs.

The new type plotter, moreover, will be used for such studies as fluid flow patterns and velocity gradients in steam and gas turbine blade grid assemblies, electrode shapes in vacuum and thyratron tube design, and field patterns in wave guides and electron lenses.

It consists basically of a plotting board and exploratory stylus, d-c. power supply operated from 115 volts a.c., a high accuracy multi-turn voltage divider, sensitive microammeter with specially designed overload protection, and a supply of conducting paper and silver paint. An instruction book accompanies the plotter. Total equipment has a shipping weight of 20 lb.

Wayne University Computing Machine Laboratory.—Grants totalling $200,000 from General Motors Corporation and the Ford Motor Company have been accepted by Wayne University in Detroit, for expansion and maintenance of its large-scale computing machine laboratory.

Wayne which now has two large-scale computing machines, a "differential analyser" and a "cinema integraph," will acquire a third with the $150,000 grant from General Motors. The machine, a digital computer, will be built by the Burroughs Adding Machine Company at the cost of labor and material and Burroughs will also assist in its design. Estimated value of the services to be contributed by Burroughs is $100,000. The computer will be the first of its kind in the middle-west and is expected to be completed within a year.

The $50,000 grant from the Ford Motor Company will be used for operating expenses for the expanded laboratory.

Wayne officials stated that the new machine would make possible a complete instructional program in large-scale machine computation. The only area where instruction in digital computation is now available is on the east coast. The officials also said that the instructional program would bring to Detroit specialists who could advise industry and business on problems dealing with the operation, application and construction of computing machines.

The first endeavor of its kind initiated without government support, the laboratory is an example of industry-education cooperation that will provide

education and training, opportunity for research, and a service to the community.

Training, especially in digital computation, is necessarily a time-consuming process, involving the mastery of new concepts and new techniques. As those machines become available for industrial use, a large number of individuals need to be trained in preparing problems for them, in their operation, as well as in designing computers more suitable for problems encountered in specific instances. As mentioned earlier, training in analogue computation is now available at Wayne University. It is urgent that this training program be enlarged to include training in digital computation in a thorough manner, to meet the growing needs of industry.

Up to now, automatic computing has been mainly a military show. With research problems growing beyond the grasp of individual experimenters and consuming the time of whole batteries of engineers, industry claims a definite need for calculators and trained personnel.

A simple example of the importance of these machines is the fact that the time-fused bombs, guided missiles and the atom bomb would have been impossible without them. They are just as important in research on problems for improvement of peace-time products. Many problems of vibration, noise and structural strength in automobiles, which, in the past, had to be solved by trial and error method on a test track may now be worked out on this machine at a fabulous savings in manpower and expense. The improvement of products and the discovery of new products will widen work opportunities and provide a better standard of living.

An explosion-proof testchamber for electrical and electronic devices is in use at Wright Air Development Center, where items under test are operated in a mixture of air and aviation gasoline. The $\frac{3}{8}$-in. steel test chamber is designed to withstand pressure of 100 psi. Various conditions of altitude, explosive concentration, and temperature can be obtained during tests. If a device operates satisfactorily without causing an explosion, it can then be subjected to an internal explosion to determine whether it can withstand that.

A new type of portable smoke generator, developed by the Army Chemical Corps and now being used in Korea, is being tested by the Department of Agriculture for possible use in protecting crops from frost. The 180-pound machine can be handled by two men and will lay a protecting layer of fog over an area five miles long and 200 yd. wide. The tests started last spring after a number of similar machines loaned by the Army Chemical Corps Depot at Dugway Proving Ground, Utah, were used successfully as smudge pots to save a multi-million dollar fruit crop during a cold snap.

A successful de-icing system for helicopter rotor blades has been announced by the Navy Bureau of Aeronautics. Developed by Bell Aircraft Corp., the de-icer consists mainly of neoprene boots covering entire length of the blades, and carrying embedded wires through which an electric current flows. In a series of 50 tests atop 6000-ft. Mt. Washington, the system operated satisfactorily in temperatures ranging from 29 degrees to minus 14 degrees F., in freezing rain, snow and winds up to 80 mph.